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The Measurement of the Eye Movements of a Prone Observer during Monocular Fixation

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ABSTRACT. A critical survey of previous work on the measurement of fixation eye movements shows that none of it has satisfied all the conditions : (i) possibility of detection of movements of magnitude one minute of arc; (ii) little interference with the natural state of the eye; (iii) satisfactory treatment of the head movement problem.

A technique which more nearly meets these requirements is described. Photoelectric recording of the movements of an ultraviolet beam reflected from the surface of the cornea enables the first and second conditions to be satisfied. The reflected beam is divided into two parts, one of which falls on a horizontal straight edge, the other on a vertical straight edge. In each case more or less of the radiation passes the straight edge as the eye moves, and is focused on an electron multiplier phototube. The output of each multiplier is amplified and fed to a cathode-ray oscillograph, the time-base of which is suppressed. The oscillograph beams are thus arranged to give vertical traces only, and these are photographed simultaneously on a continuously moving film travelling in the horizontal direction. The eye movements can be deduced from the two records on the film. In an attempt to meet the third requirement the subject is placed in the prone position, in which the movements of the ultraviolet beam due to head movements are smaller than for other positions of the subject.

No attempt at analysis of the observations has yet been made.

§ 1. INTRODUCTION

THE measurement of fixation eye movements is of importance because an adequate theory of many visual phenomena must take into account any movements of the retinal mosaic over the image formed by the eye during fixation. Fixation eye movements have been studied by many previous workers, but the methods leave much to be desired ; the treatment of the head movement problem is, in general, inadequate and high sensitivity has been obtained at the expense of introducing foreign bodies on to the surface of the eye.

§ 2. EXPERIMENTAL

The requirements of an improved technique were taken as : (i) possibility of detection of movements of magnitude one minute of arc ; (ii) little interference with the natural condition of the eye ; (iii) satisfactory treatment of the head movement problem.

In a preliminary investigation, the deflection of a beam of light reflected from a plane mirror attached to a spectacle frame worn by the subject was used as an indication of the head movements. It was found that when the subject was sitting up, whatever the support used for the head, the movements were about ten times greater in magnitude than those of the subject in the prone position biting on a dental impression. This position was therefore adopted. The subject's head rested on a firm foundation of variable height and was partially supported in a sling (figure 1). For convenience, most other parts of the equipment, the lay-out of which is indicated in figure 2, were mounted on the floor.

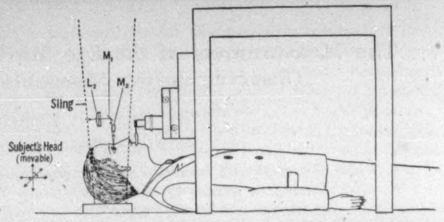

Figure 1. The subject in position.

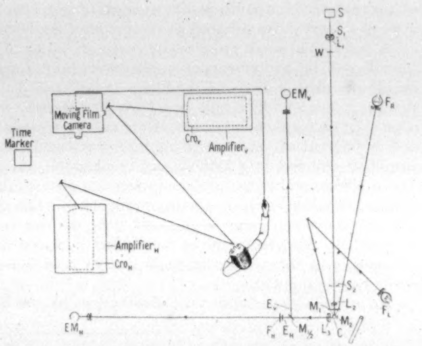

Figure 2. The lay-out of the apparatus.

The principle of the system is to direct a beam of ultraviolet light ($\lambda=3650$ A.) on to the cornea C in such a direction that the fraction of the light transmitted through the cornea falls on the subject's blind spot when some test object is fixated on the fovea with the illuminated eye: the subject is then, in effect, only aware of the fixation target. Part of the radiation is, however, reflected at the cornea; this reflected beam strikes a half-aluminized mirror ($M_{\frac{1}{2}}$) and is there divided so that one part falls on a horizontal straight edge (E_H) and the other on a vertical one (E_V). In each case more or less radiation passes the straight edge as the eye moves; the fraction passed is focused on an electron multiplier photocell (EM_H for the transmitted, EM_V for the reflected beam). The output of each multiplier is amplified and fed to a cathode-ray oscillograph (with blue screen), the time-base of which is suppressed. The two oscillograph beams are in this way arranged to give vertical traces only which are photographed simultaneously on a continuously moving film travelling in the horizontal direction. The eye movements can be deduced from the two records on the film.

The various parts of the apparatus will now be described in detail.

(i) *The optical system*

The corneal illumination is obtained from a 250-watt high pressure mercury vapour lamp S operating from car batteries. S is imaged by the lens L_1 (situated immediately in front of the square aperture S_1 of side 2 mm.) in the plane of the square aperture S_2 of side 8 mm.; S_2 is situated at the focus of the lens L_2. After passing through the lens L_2, the light is reflected at the subject's cornea by the plane aluminized mirror M_1 inclined at approximately 45° to the vertical. The incident radiation was largely confined to $\lambda 3650$ A. by the Wood's glass filter W; the maximum value of its energy is about 3 μw. The part of the ultraviolet beam reflected at the cornea strikes the plane steel mirror M_2 (inclined at about 45° to the vertical and at 90° to M_1) centrally, and is there reflected along the axis of L_3 to M_1. The angle of incidence at the cornea is about 3°. The illumination in the plane of the straight edges is sensibly uniform since they are imaged on S_1 by the lens L_3, the subject's cornea and the lens L_2. The position occupied by the beam under these conditions is marked by spots of phosphorescent paint on a fluorescent screen (F_H) which can be swung into position immediately behind E_H when required.

R.C.A. 931 A. multipliers were used since, as far as is known, they are the most sensitive available; the tubes employed were specially selected for minimum fatigue. From knowledge of the sensitivity distribution of the photo-cathode, the most suitable lens systems for condensing the ultraviolet beam on the multipliers were calculated and the best from those available selected by trial: a sphero-cylindrical lens giving an image 2 mm. long and ¼ mm. wide on the multiplier cathode was used in each case. The multiplier response was then sensibly uniform for positions of the cornea in a 5 mm. range around that for which S_1 is imaged on the straight edges; the size of the patch on the multiplier was, moreover, independent of the position of the straight edges.

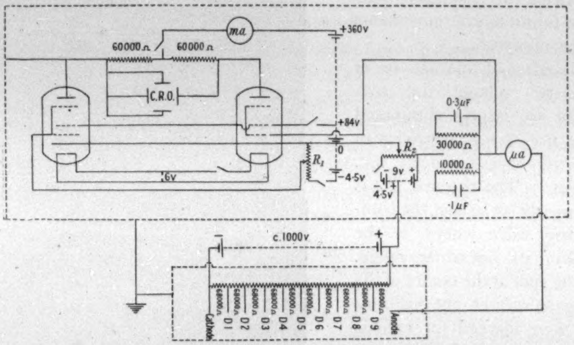

Figure 3. Circuits used with each multiplier.

The system was calibrated by using a lens with the same curvature as that of the cornea. The straight edges were set so that each cut off approximately half the patch when this was central. The responses of EM_H and EM_V were then recorded for complete movement of the beam over the straight edges for each of a series of vertical positions of the "artificial cornea" 1 mm. apart and lying in the 5 mm. range surrounding the position for which S_1 is imaged on the straight edges. Apart from the initial and end portions all the curves were linear. The sensitivity reaches a maximum value (3·7 μa/min.) for the position in which S_1 is imaged on the straight edges and falls to about 3 μa/min. at either end of the 5 mm. range.

The mirror M_1 also served to reflect the fixation beam into the eye. To enable observations to be made on either eye, two fixation targets were available. Each was illuminated by a filament lamp: F_R and F_L for the right and left eyes respectively. For subjects with refractive errors, correcting lenses were inserted in the path of each beam ; neutral and coloured filters could also be inserted.

(ii) *The electrical system*

The lay-out of the circuits used in conjunction with either multiplier is shown in figure 3 ; each circuit is mounted in two earthed tin boxes as indicated by the dotted lines. The signal fed to the amplifier is obtained by passing the multiplier output through a resistance (either 10 or $30 \times 10^3 \Omega$). The purpose of the condenser connected across this resistance is to suppress the major portion of the background noise from the multiplier; the cut-off frequency of the amplifier is then about 1000 c/sec. There is therefore little attenuation of frequencies of 100 c/sec.—the highest found in eye movements by other observers.

The effect of fluctuations in the heater voltage and H.T. supplies are largely eliminated by the use of H.F. pentodes in the single-stage balanced D.C. amplifier. The rheostat R_1 is permanently set so that the compensating valve works at the centre of its operating range. With the spot at the centre of the oscillograph screen, the positions of R_2 were marked for various microammeter readings, for low and high gain; change from one position to another could then easily be made. The amplifiers were found to have good stability: in figure 4 (*a*) is a record of the performance for no signal from the multipliers. The amplifiers are, moreover, sensibly linear; from the relationship between microammeter reading and screen deflection, it appears that the maximum value of the sensitivity in either direction is about 6 mm. screen deflection per minute; deflections due to noise from the multipliers do not, in general, correspond to more than $\frac{1}{2}$ minute of arc; a record from the stationary artificial cornea is shown in figure 4 (*b*).

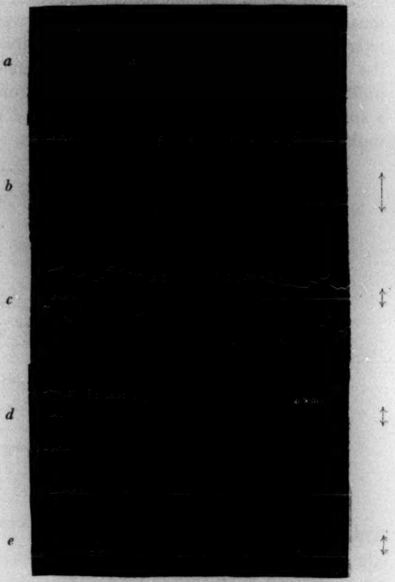

Figure 4.

(*a*) Amplifier performance for no signal from multipliers.
(*b*) Record from stationary artificial cornea.
(*c*) Eye movements during fixation.
(*d*) Head movements.
(*e*) Record from an eye in the absence of the straight-edges.

Time marked by dots, 50/sec.; amplitude shown at side : length of line between arrow heads represents 10 minutes of arc.

(iii) *Positioning the subject*

As a first approximation, the rod R is swung over the eye under investigation : when the head height is correct, the cork-covered peg attached to R just touches the eyelid. After a few minutes in the dark, the subject is asked to bite on the dental impression and look for a blue haze—a fluorescence effect in the eye due to the ultraviolet irradiation. To facilitate the fine adjustment of the height of the head, the square S_2 is diaphragmed into an L and the head height adjusted until the haze appears square and as small as possible : when the head is too high, the haze appears as an upright L ; when it is too low, as an inverted L. The beam is observed on F_H and the subject's head moved in the horizontal plane until the patch is central. The subject is then asked to observe the position of the blue haze when looked at directly relative to the fixation target and the blue and red images of S_2. The straight edges are then set in the appropriate position as marked by spots of phosphorescent paint ; movements of the eye from side to side produce changes in the amount of light passing E_H ; those up and down the length of the observer cause changes in the amount passing E_V.

§ 3. RESULTS

The results so far have been confined to the investigation of the possibilities of the method. It appears that eye movements of appreciable magnitude may sometimes be present. Figure 4 (*c*) shows a portion of the record of an observer for fixation of the centre point of a cross, angular length of arm 20′, width 1′. Figure 4 (*d*) shows the effect of head movements on those of the corneal reflex obtained by supporting the artificial cornea on a spectacle frame immediately above the live one. By calculation, the corresponding compensatory eye movements are negligible. In figure 4 (*e*), the record obtained from a live cornea in the absence of the straight edges is shown.

§ 4. DISCUSSION

It appears from the preliminary records so far obtained, that the method satisfies the three requirements laid down at the beginning of the experimental section more closely than any previously used. Simultaneous recording of the artificial corneal reflex would be a desirable feature and it is hoped to make arrangements for this. It may be argued that most visual characteristics involving eye movements have been investigated under conditions different from those described in this paper and that such measurements might be irrelevant to those problems. Whether or not this is a valid criticism will appear in the course of the investigations since it is anticipated that these will include the study of the various characteristics under the conditions described.

ACKNOWLEDGMENTS

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On the Designing of Aspheric Surfaces

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ABSTRACT. A method is developed for the design of an aspheric surface to ensure axial stigmatism in any given centred optical system. It is shown that when the surface is in the interior of the system the determination of its profile depends on the solution of a certain transcendental equation. An iterative method is described which gives the profile to any desired degree of accuracy. When the corrector is the last surface of the system the solution is greatly simplified and for such cases exact parametric equations, involving quantities which can easily be obtained from a ray trace, are given.

§ 1. INTRODUCTION

THE excellent performance of the Schmidt camera, as well as the increasing demand for improved image qualities of optical systems, has led in recent years to the construction of a number of systems which employ aspheric surfaces. This requires the development of methods which allow the designer to determine what shapes such surfaces are to take in order to obtain good performance over a finite field. With the exception of the elegant "see-saw" diagram of Burch (1942) no method is at present available for dealing with this question. Burch's method is restricted to the extended paraxial region, i.e. the region in which aberrations above the third order (i.e. aberrations higher than Seidel's) can be neglected. This method enables us to find the most favourable range of the design parameters and is particularly useful in the early stages of the design. In systems where higher aberrations have to be taken into account the Seidel design needs to be supplemented by a further stage of approximation, involving the modification of one or more surfaces of the system.* Although in general one aims at the optimum performance over the whole of a finite field, it is often sufficient to design the correcting surface in such a way that a particular aberration is corrected at one point of the field.

The present paper is concerned with the design of a correcting surface ensuring axial stigmatism in any given centred system. Methods which deal with this question have been previously described by Linfoot (1943), Luneberg (1944), Herzberger and Hoadley (1946) and Wolf and Preddy (1947).

The method of Linfoot is suitable for the design of two aspheric surfaces in certain astrographic cameras, ensuring both the absence of axial spherical aberration and the satisfaction of the exact sine condition. The method is still applicable if we aim only at the design of a correcting plate to attain axial stigmatism in systems where, as in the Schmidt and Meniscus-Schmidt cameras and Schmidt-Cassegrain systems, the corrector is the first or last surface.

Whenever the aspheric surface is adjacent to one of the foci its profile may be determined by the exact equations of Luneberg (related to equation (4.6) of the present paper) which he obtained by the use of characteristic functions.

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* This problem is examined by D. S. Volosov (*J. Opt. Soc. Amer.*, 1947, **37,** 342).

A method with a wider range of application was developed by Herzberger and Hoadley. It can be used for the design of a correcting surface which is situated anywhere in the system. This method is based on the computation of a number of approximate surfaces, which approach the ideal.

More recently, Preddy and the writer have shown how to design a plano-aspheric lens which would give axial stigmatism to a given symmetrical pencil of rays. The present paper gives an extension of our analysis which applies to the design of a correcting surface, whether or not it is situated in the interior of the system. The computation is based on exact equations for the surface in question. These equations involve quantities which can be easily obtained from a ray trace. In some cases the use of iterative methods is required, but the accuracy reached at every stage may be estimated without the additional ray traces required in the method of Herzberger and Hoadley. When the corrector is adjacent to one of the foci no iteration is needed, and for such cases exact parametric equations are given.

§2. PRELIMINARY RESULTS

The solution of our main problem will be based on some properties of normal rectilinear congruences (N.R.C.) which we discuss in this section. For the purpose of dealing with axially symmetrical systems, it is sufficient to consider the case of two dimensional N.R.C.s.

(i) Let Γ be a plane N.R.C. of rays. We choose rectangular axes OX, OY and denote by h the height at which a typical member of Γ meets OY, or would meet OY if produced sufficiently far. Further, we denote by $\omega\,(-\pi<\omega\leqslant\pi)$ the angle which this typical ray makes with OX; we measure ω anticlockwise from the positive x direction to the direction along which the light advances. The congruence is then completely specified by a functional relation between ω and h say

$$\omega=\omega(h) \qquad \ldots\ldots(2.1)$$

where h is regarded as a free parameter. We shall refer to a ray, determined by the value h_r of the parameter h, as the ray h_r and denote by ω_r the corresponding value of ω. The following simple theorem can then be proved * by differential geometry or otherwise:

Theorem A: If two rays h_1 and h_2 intersect a wave front W of Γ in P_1 and P_2, and OY in Q_1 and Q_2 (figure 1), then the optical path difference $[P_2Q_2]-[P_1Q_1]$ is given by

$$[P_2Q_2]-[P_1Q_1]=n\int_{h_1}^{h_2}\sin\omega\,dh \qquad \ldots\ldots(2.2)$$

where n is the refractive index of the medium in which the rays are situated.

(ii) With the help of theorem A we now examine the (1, 1) correspondence between the parameters of two N.R.C.s which are transformed into each other by refraction or reflection. In general the possibility of such a transformation is asserted by the converse of Malus' theorem (Levi-Civita 1900).

Let Γ and Γ' be two coplanar N.R.C.s, situated in spaces of respective refractive indices n and n', and let Σ be a refracting (or reflecting) profile which transforms these two congruences into each other. To simplify the analysis we refer Γ and Γ' to the same Cartesian axes with their origin coincident with a point of Σ. Let h_r and h_r' be any two corresponding rays in this transformation; that is to say, the ray h_r of Γ is transformed by refraction at Σ into the ray h_r' of Γ' (figure 2). Further, let W and W′ denote wave fronts of Γ and Γ' respectively. Also, let P_r and Q_r be the points of intersection of the ray h_r with W and OY respectively, and similarly for the dashed letters. (By definition $OQ_r = h_r$, $OQ_r' = h_r'$). Finally let (x_r, y_r) denote the coordinates of the point R_r of Σ where these rays meet.

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* See for example Wolf and Preddy (1947), p. 705. It was assumed there that the congruence is symmetrical and $n=1$; but it can easily be verified that these restrictions are not necessary. The difference in signs of the integrals is due to a different sign convention used in the two papers.

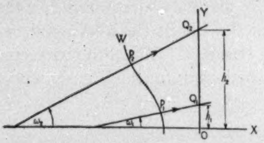

Figure 1.

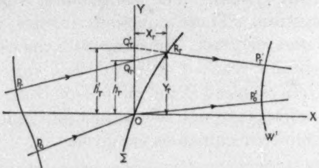

Figure 2.

Since the optical path between the two wave fronts is constant when taken along corresponding rays,

$$[P_rR_r] + [R_rP_r'] = [P_0O] + [OP_0'], \qquad \ldots\ldots(2.3)$$

where the square brackets denote optical lengths and the suffix zero refers to rays which pass through O.

We have from figure 2 and theorem A, that

$$[P_rR_r] = [P_rQ_r] + [Q_rR_r] = [P_0O] + n\int_0^{h_r} \sin\omega\, dh + nx_r \sec\omega_r \qquad \ldots\ldots(2.4)$$

and similarly,

$$[R_rP_r'] = -[P_0'O] - n'\int_0^{h_r'} \sin\omega'\, dh' - n'x_r \sec\omega_r'. \qquad \ldots\ldots(2.5)$$

Substituting from (2.4) and (2.5) into (2.3), we obtain

$$x_r(n\sec\omega_r - n'\sec\omega_r') + n\int_0^{h_r} \sin\omega\, dh - n'\int_0^{h_r'} \sin\omega'\, dh' = 0 \quad \ldots\ldots(2.6)$$

We also have from figure 2

$$y_r = h_r + x_r\tan\omega_r \qquad \ldots\ldots(2.7)$$

and

$$y_r = h_r' + x_r\tan\omega_r'. \qquad \ldots\ldots(2.8)$$

On eliminating x_r and y_r between (2.6), (2.7) and (2.8) we obtain the following interesting invariant property :

Theorem B : In the transformation $\Gamma \to \Gamma'$, the parameters of corresponding rays, when referred to any set of rectangular axes with their origin placed at a point of the transforming profile, satisfy the relation

$$\sin(\omega_r - \omega_r')\left(n\int_0^{h_r} \sin\omega\, dh - n'\int_0^{h_r'} \sin\omega'\, dh'\right) + (n\cos\omega_r' - n'\cos\omega_r)(h_r' - h_r) = 0$$

$$\ldots\ldots(2.9)$$

§3. THE DESIGN OF A CORRECTING SURFACE ACHIEVING AXIAL STIGMATISM

(i) In a number of optical systems employing an aspherical surface, the latter is designed to ensure axial stigmatism. Examples of this are the Schmidt television projector system, certain modern astrographic cameras and the semi-aplanatic reflecting microscope. With the help of the results of the previous section, we shall now show how to determine the profile of such a correcting surface for any given centred optical system.

We introduce a rectangular reference system with origin O at the pole (assumed to be known) of the correcting surface Σ and with OX along the axis of symmetry. Rays starting from the axial object point S are refracted or reflected at the intermediate surfaces and form an N.R.C. Γ in the space preceding Σ. In figure 3 a typical member of this congruence is denoted by TR. Let

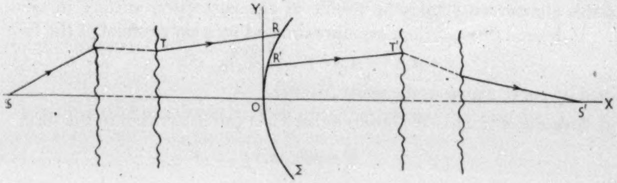

Figure 3.

$$\omega = \omega(h) \qquad \ldots\ldots(3.1)$$

be the equation of this congruence. In practice, it is often more convenient to use instead of (3.1) a polynomial approximation of the type

$$\sin \omega = a_1 h + a_3 h^3 + \ldots\ldots a_{2k+1} h^{2k+1}. \qquad \ldots\ldots(3.2)$$

This is obtained by tracing a number of rays from the axial object point to the space which precedes Σ, computing for each ray the quantities ω and h and then fitting a polynomial in the usual manner. The use of such an approximation, however, is not essential, and when great accuracy is required may not even be desirable. In such cases it may be better to use a table of values of ω with corresponding values of h based on the tracing of a thick fan of rays.

In the space which follows Σ, the rays which are traced backwards * from the ideal axial image point S′, will form another N.R.C. Let Γ′ be the congruence obtained from this " backtraced " congruence, by reversing the direction of every ray, i.e. we regard the rays of Γ′ as proceeding from the correcting surface towards the image point. For Γ′ we again obtain a relation of the type (3.1) or (3.2). We shall use dashed letters to refer to all quantities related to Γ′.

It is clear that the required surface will transform Γ into Γ′. Hence it follows from theorem B that the parameters of corresponding rays in this transformation satisfy the relation

$$\sin(\omega_\mathrm{r} - \omega'_\mathrm{r}) \left(n \int_0^{h_\mathrm{r}} \sin \omega \, dh - n' \int_0^{h'_\mathrm{r}} \sin \omega' \, dh' \right) + (n \cos \omega'_\mathrm{r} - n' \cos \omega_\mathrm{r})(h'_\mathrm{r} - h_\mathrm{r}) = 0. \qquad \ldots\ldots(3.3)$$

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* When the correcting surface is the last (or first) surface of the system, the backwards ray trace becomes superfluous. The simplified analysis for such cases is given in § 4.

This relation enables us to compute the profile of Σ. For if h_{r} is any particular ray of the first congruence, the corresponding h'_{r} can be obtained by solving (3.3) as a transcendental equation (§ 3 (ii)); the coordinates of the point $(x_{\mathrm{r}}, y_{\mathrm{r}})$ on Σ are then given by the equations (2·6) and (2·7), which can be written as

$$x_{\mathrm{r}}+iy_{\mathrm{r}}=\frac{\cos\omega'_{\mathrm{r}}\exp(i\,\omega_{\mathrm{r}})}{n\cos\omega'_{\mathrm{r}}-n'\cos\omega_{\mathrm{r}}}\left(n'\int_0^{h'_{\mathrm{r}}}\sin\omega'\,dh'-n\int_0^{h_{\mathrm{r}}}\sin\omega\,dh\right)+ih_{\mathrm{r}}. \quad \ldots\ldots(3.4)$$

Although it may be simpler to evaluate x_{r} and y_{r} by the use of some other combination of the relations (2.6), (2.7) and (2.8), it is better to use the above form; for any error arising in an approximate solution of (3.3) will cause as a rule smaller errors in the coordinates when these are calculated from (3.4) than in the other cases. This is easily verified by examining the Taylor's expansions of the various combinations.

In this manner we obtain the profile of the correcting surface in tabulated form. If desired this can then be approximated by a polynomial of the form

$$x=A_2y^2+A_4y^4+\ldots.+A_{2m}y^{2m} \quad \ldots\ldots(3.5)$$

obtained by curve fitting in the usual manner.

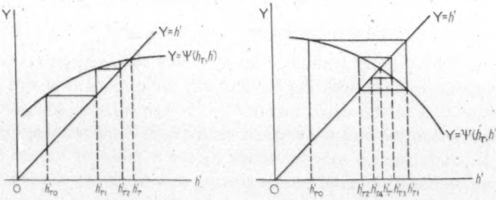
(ii) *Solution of* (3.3). We observe that (3.3) can be written in the form

$$h'_{\mathrm{r}}=\psi(h_{\mathrm{r}}, h'_{\mathrm{r}}) \quad \ldots\ldots(3.6)$$

where

$$\psi(h_{\mathrm{r}}, h'_{\mathrm{s}})=h_{\mathrm{r}}+\frac{\sin(\omega_{\mathrm{r}}-\omega'_{\mathrm{s}})}{n\cos\omega'_{\mathrm{s}}-n'\cos\omega_{\mathrm{r}}}\left(n'\int_0^{h'_{\mathrm{s}}}\sin\omega'\,dh'-n\int_0^{h_{\mathrm{r}}}\sin\omega\,dh\right) \quad \ldots\ldots(3.7)$$

and ω_{r} and ω'_{s} are connected with h_{r} and h'_{s} respectively by relations of the type (3.1) or (3.2).



(*a*) $0<\frac{\partial\psi}{\partial h'}<1$. (*b*) $-1<\frac{\partial\psi}{\partial h'}<0$.

Figure 4. Illustrating the convergence of sequence (3.8).

Suppose we take an approximation (say $h'_{\mathrm{r},0}$) to h'_{r} given h_{r}. We then form a sequence

$$h'_{\mathrm{r},0},\ h'_{\mathrm{r},1},\ h'_{\mathrm{r},2}\ \ldots.. \quad \ldots\ldots(3.8)$$

where

$$h'_{\mathrm{r},i+1}=\psi(h_{\mathrm{r}}, h'_{\mathrm{r},i}). \quad \ldots\ldots(3.9)$$

Such a sequence will converge to the desired root h'_{r} of (3.3) provided that, in the neighbourhood of this root,

$$|\partial\psi/\partial h'_{\mathrm{s}}|<1. \quad \ldots\ldots(3.10)$$

The geometrical interpretation of the successive values of (3.8) is shown in figures 4 (*a*) and 4 (*b*); it is seen that the rapidity of convergence depends on the magnitude of $\partial\psi/\partial h'_s$. We find on differentiating (3.7) that

$$\frac{\partial\psi}{\partial h'_s}=\left(\frac{d\omega'_s}{dh'_s}\right)\left(n'\int_0^{h'_r}\sin\omega'\,dh'-n\int_0^{h_r}\sin\omega\,dh\right)\frac{\cos\omega_r\,[n'\cos(\omega_r-\omega'_s)-n]}{[n\cos\omega'_s-n'\cos\omega_r]^2}+\frac{n'\sin\omega'_s\sin(\omega_r-\omega'_s)}{[n\cos\omega'_s-n'\cos\omega_r]}.\qquad\ldots\ldots(3.11)$$

As a rule, ω, ω', and $d\omega'/dh'$ will be small* so that (3.10) is satisfied and the convergence of the iteration is rapid.

Suppose now that we stop at a particular term (say h'_{rt}) in (3.8) and use it in (3.4) for the calculation of x_r and y_r. We can make a rough estimate of the errors Δx_r and Δy_r thus introduced. We find, on expanding (3.4) and neglecting second and higher powers, that

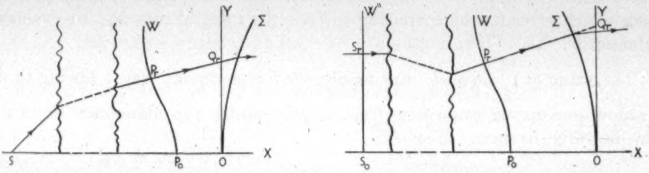
$$\Delta x_r=O(\omega'_{rt}\,\Delta h'_{rt});\quad \Delta y_r=O(\omega_r\omega'_{rt}\Delta h'_{rt})\qquad\ldots\ldots(3.12)$$

where the difference $\Delta h'_{rt}$ defined by

$$\Delta h'_{rt}=h'_r-h'_{rt}\qquad\ldots\ldots(3.13)$$

can be estimated from the behaviour of the sequence.

The above estimates show that even a rough approximation to h'_r will often give a good approximation to x_r and y_r.†



(*a*) $\left\{[SQ_r]-[SO]\right\}\Big/n=\int_0^{h_r}\sin\omega\,dh.$ (*b*) $\left\{[S_rQ_r]-[S_0O]\right\}\Big/n=\int_0^{h_r}\sin\omega\,dh.$

Figure 5.

The initial value $h'_{r,0}$ in (3.8) can be chosen in various ways. We can, for example, take as $h'_{r,0}$ the value of the parameter which corresponds to h_r in the congruence obtained by the refraction of Γ in the *uncorrected* surface. But even the rough approximation $h'_{r,0}=h_r$ will be adequate in most cases.

(iii) *Evaluation of* $\int_0^{h_r}\sin\omega\,dh$. The integral $\int_0^{h_r}\sin\omega\,dh$ which occurs in the previous analysis can be evaluated in several ways.

If the polynomial approximation (3.2) is used, the integral is given at once by

$$\int_0^{h_r}\sin\omega\,dh=\frac{a_1}{2}h_r^2+\frac{a_3}{4}h_r^4+\ldots\ldots\frac{a_{2k+1}}{2k+2}h_r^{2k+2}.\qquad\ldots\ldots(3.14)$$

There are other methods which do not depend on an expansion for $\sin\omega$.

---

* The term $d\omega'_s/dh'_s$ has the following interpretation: if ρ'_s denotes the radius of curvature of the wave front of Γ' which passes through the point $(0, h'_s)$ then $d\omega'_s/dh'_s=(\cos\omega'_s)/\rho'_s$.

† For example, in a Meniscus-Schmidt Projector (with the corrector facing the meniscus) of short focus 8·7 in., plate diameter 10·3 in. and throw distance 128 in., the approximation $h'_r=h_r$ gave an accuracy of 1/10 of a refraction fringe in the coordinates of the plate profile. It is generally possible thus to avoid the iteration with small loss of accuracy if the corrector is a plate.

Let S denote the axial object point. If P_r and P_0 (figure 5 (*a*)) lie on the same wave front W, in the space which precedes Σ we have

$$[SP_r]=[SP_0]. \qquad \ldots\ldots(3.15)$$

From theorem A and (3.15) it follows that

$$\int_0^{h_r} \sin \omega \, dh = \frac{1}{n}\left\{[SQ_r]-[SO]\right\}. \qquad \ldots\ldots(3.16)$$

Similarly, if the axial object point is at infinity, we have

$$\int_0^{h_r} \sin \omega \, dh = \frac{1}{n}\left\{[S_rQ_r]-[S_0O]\right\}. \qquad \ldots\ldots(3.17)$$

The meaning of S_r and S_0 is shown in figure 5 (*b*), where W″ is any line perpendicular to OX in the object space.

Equation (3.16) [or (3.17) when the object point is at infinity] gives the integral in terms of an optical path difference which can be obtained from ray trace. We found, however, in numerical applications that there was a large drop of significant figures when the integral was evaluated in this way, so that in such cases it may be advisable to use a larger number of figures in the computation of the paths of the rays.

If a sufficient number of rays are traced from the object space, up to the space which precedes the correcting surface, the integral may also be evaluated numerically.

The value of $\int_0^{h'_r} \sin \omega' \, dh'$ can be obtained in a similar way. Owing to the eration process an evaluation from a polynomial approximation for $\sin \omega'$ may, however, be more convenient.

§4. SPECIAL CASE: THE DESIGN OF A CORRECTOR WHICH IS THE FIRST OR LAST SURFACE IN A SYSTEM

(i) In practically all aspheric systems which have been constructed up to now, the corrector is the first or last surface in the system. The selection of the surface which is to be changed into a corrector depends of course primarily on the effect which the latter will have on the off-axis performance of the system. Cooke triplets of large aperture-ratio, for example, suffer badly from zonal aberration. It was thought at first that the figuring of the central concave lens, which in most designs is at or near the stop, would remove this without changing seriously the off-axis errors. Higher coma was, however, in consequence considerably aggravated, presumably because the correction increased the departure from the exact sine condition. It was found better to figure the last surface and to rebalance the coma by slightly modifying the design.

When the corrector is the last surface of the system the computation of its profile is considerably simplified. In such a case the congruence Γ′ is given by

$$\tan \omega' = (\text{constant}) \,.\, h' \qquad \ldots\ldots(4.1)$$

and the equations of the corrector can be obtained from our previous analysis by eliminating ω' and h' between (3.3), (3.4) and (4.1). As the elimination is rather laborious, we shall deduce the solution directly.

Let Γ be the congruence in the space which precedes the last surface, and let S′ be the ideal axial image point to which all the rays are to converge after refraction at this surface. We denote by s' the distance OS′ measured along the positive direction of the axis and retain the previous symbols for the other quantities (figure 6). We drop the suffix r as no confusion is likely to occur.

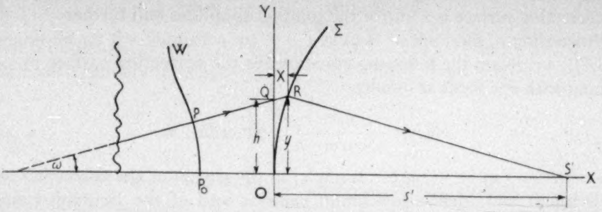


Figure 6.

Since the optical path from the wave front W to S′ is constant

$$[PQ] + nx \sec \omega + n'[(s' - x)^2 + (h + x \tan \omega)^2]^{1/2} = [P_0O] + n's'. \qquad \ldots\ldots(4.2)$$

From (4.2) and theorem A it follows that

$$n\int_0^h \sin \omega \, dh + nx \sec \omega + n'[(s' - x)^2 + (h + x \tan \omega)^2]^{1/2} - n's' = 0. \qquad (4.3)$$

Rationalizing (4.3), we find that x satisfies the equation

$$Ax^2 + 2\,Bx \cos \omega + C \cos^2 \omega = 0 \qquad \ldots\ldots(4.4)$$

where
$$A = n^2 - n'^2\,;$$
$$B = n^2\int_0^h \sin \omega \, dh - nn's' + n'^2\,(s' \cos \omega - h \sin \omega)\,;$$
$$C = \left(n\int_0^h \sin \omega \, dh\right)\left(n\int_0^h \sin \omega \, dh - 2n's'\right) - n'^2h^2.$$

Solving (4.4) and using the relation

$$y = h + x \tan \omega \qquad \ldots\ldots(4.5)$$

we finally obtain the following *exact equation for the correcting surface* in terms of the free parameter h:

$$x + iy = -(e^{i\omega}/A)[B \pm (B^2 - AC)^{1/2}] + ih. \qquad \ldots\ldots(4.6)$$

Using the boundary condition $x = y = 0$ when $h = \omega = 0$, we find that the positive or negative sign of the square root is to be taken according as $n' < n$ or $n' > n$. The integral which occurs in B and C can again be evaluated by one of the methods discussed in § 3 (iii).

In order to avoid confusion about the value of ω, we emphasize that *in applying* (4.6) *or any other equation of* § 4 *we regard the rays of the congruence* Γ *as proceeding towards the correcting surface*. Thus, for instance, in the Meniscus-Schmidt system (figures 8 (*a*), 8 (*b*)), when calculating the correcting surface, we interchange the object and image points. In figure 7, which shows a Schmidt projector suitable for cathode tube projection a typical member of the congruence Γ is denoted by TR; h is then the height at which TR produced meets OY. The corresponding value of ω lies between $-\pi/2$ and 0.

(ii) *Refracting surfaces*, $s' = \infty$. When one of the foci is at infinity or when the correcting surface is a mirror the solution simplifies still further.

Proceeding to the limit of (4.6) as $s' \to \infty$ (or setting $\omega' = 0$ for all values of h' in (3.4)), we obtain the following equation for the correcting surface in systems working with one focus at infinity :

$$x + iy = \frac{n e^{i\omega}}{n' \cos\omega - n} \int_0^h \sin\omega \, dh + ih. \qquad \ldots\ldots(4.7)$$

This equation can be used, for example, in the design of the correcting plate of the Schmidt and Meniscus-Schmidt cameras and of the Schmidt-Cassegrain systems. Clearly (4.7) applies whether the aspheric or the plane side of the plate faces the incoming light. The two cases for the Meniscus-Schmidt camera of Hawkins and Linfoot (1945) are illustrated in figures 8 (*a*) and 8 (*b*). As before, a typical member of the congruence Γ (which of course is different in the two cases) is denoted by TR.

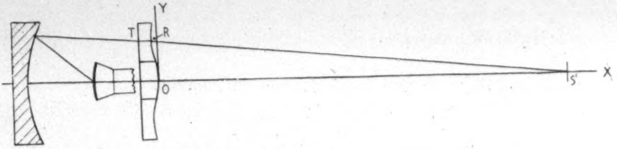
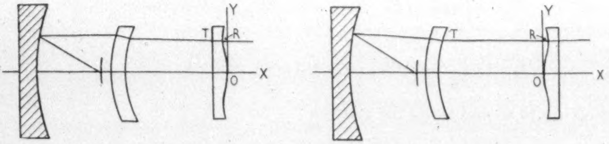


Figure 7. Schmidt projector system.



(*a*) $s' = \infty$, $n = n_{\text{plate}}$, $n' = 1$. (*b*) $s' = \infty$, $n = 1$, $n' = n_{\text{plate}}$.

Figure 8. Meniscus-Schmidt systems.

(iii) *Reflecting surface*, $s' < \infty$. If the correcting surface is a mirror we obtain with the same sign convention and by analysis similar to that of § 4 (i) the following exact parametric equations for its profile

$$x + iy = Ce^{i\omega}/2B + ih \qquad \ldots\ldots(4.8)$$

where
$$B = \int_0^h \sin\omega \, dh + s'(\cos\omega - 1) - h \sin\omega;$$

$$C = \left(\int_0^h \sin\omega \, dh \right) \left(2s' - \int_0^h \sin\omega \, dh \right) + h^2.$$

This case covers, for example, the semi-aplanat reflecting microscope recently constructed by Burch (1947); its optical system is shown in figure 9.

(iv) *Reflecting surface*, $s' = \infty$. Proceeding to the limit of (4.8) as $s' \to \infty$, we obtain the following equations for a correcting mirror in systems working with one focus at infinity:

$$x+iy=\frac{e^{i\omega}}{\cos\omega-1}\int_0^h \sin\omega\,dh+ih. \qquad \ldots\ldots(4.9)$$

This case is illustrated in figure 10 by a modified Cassegrain telescope with figuring on the concave primary mirror.

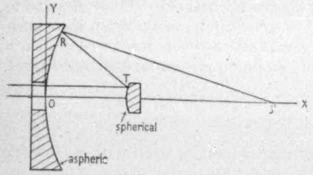


Figure 9. Semi-aplanat reflecting microscope.

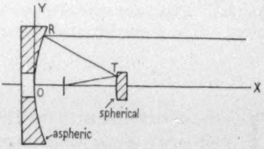


Figure 10. Modified Cassegrain telescope.

It will be noted that (4.6), (4.7), (4.8) and (4.9) are exact parametric equations of Hamilton's focal refractors and reflectors (Hamilton 1931, Synge 1937), with focus S′ on the axis of an arbitrary symmetrical congruence Γ defined by the relation $\omega=\omega(h)$.

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The Interference Pattern of Two Neighbouring Radiations in a Hilger Prism Interferometer

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ABSTRACT. When two neighbouring radiations are applied to a Hilger prism interferometer, an interference pattern of two sets of fringes is obtained. These sets can be arranged at different orientations, when a system of grey bands results from their intersection. Characteristics of these bands are described, and a theoretical consideration is given. The possibility of applying the effect to the resolution of spectral lines is discussed.

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§ 1. INTRODUCTION

In the optical industry the Hilger interferometer is usually used with a Hewittic mercury vapour lamp together with a suitable filter, so that substantially monochromatic light is used corresponding to the predominant radiation 5461 A. The interference fringes employed are similar to Newton's rings. The present paper deals both experimentally and theoretically with the effects produced when two neighbouring radiations, such as those of the sodium yellow doublet, are employed.

§ 2. EXPERIMENTAL

A Hilger prism interferometer with mirrors 85 mm. square and a large flint prism of 30° angle was used, the arrangement being indicated in figure 1. For the light source S a hot cathode sodium lamp was employed in general, but occasionally it was replaced by a mercury vapour lamp with an absorption cell filled with potassium bichromate solution. The sodium yellow doublet has a separation of about 6 A. and the mercury doublet a somewhat greater separation of 21·1 A.

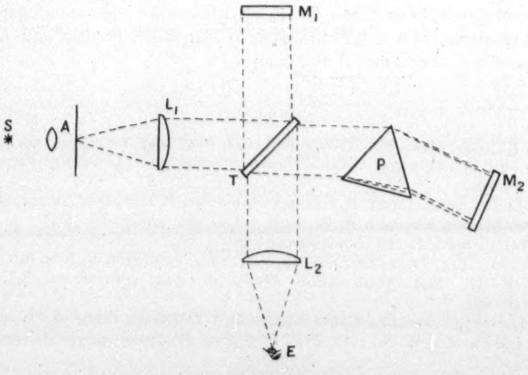
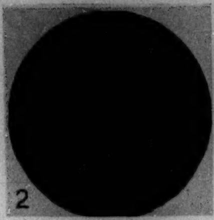


Figure 1.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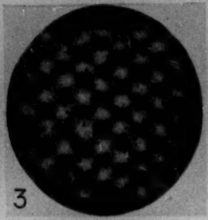
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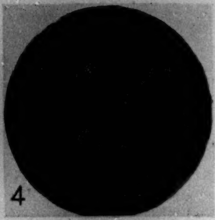
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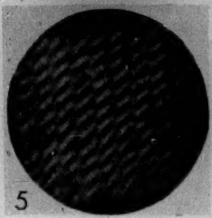
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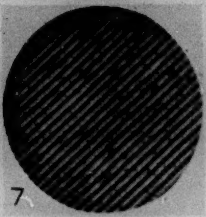
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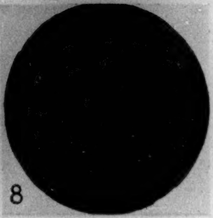
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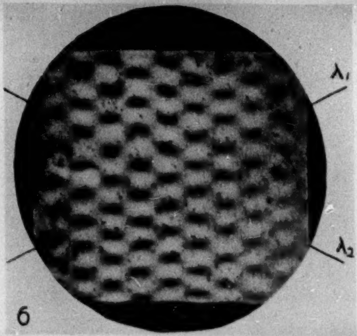
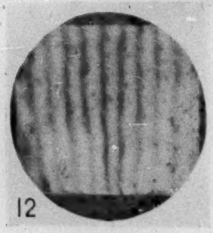

λ_1
λ_2
6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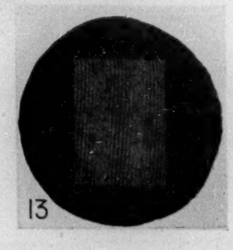
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Figures 2–8 and 12 and 13.

Prism P is arranged at minimum deviation with respect to the rays emerging from the half-silvered plate T; the paths of the rays are indicated by the dotted lines. The initial adjustment is effected by viewing with an eyepiece the various reflected images of A formed in the focal plane E of lens L_2: a single bright image is seen due to reflection from mirror M_1, and a series of coloured images due to dispersion by prism P and reflection from mirror M_2. By means of the mirror-adjusting screws the image from M_1 can be made to coincide with any one of these coloured images; with sodium light the two yellow images, from M_1 and M_2 respectively, are superimposed. Final adjustment is made in the usual way by placing the eye directly at E and moving M_1 longitudinally until the two paths are substantially equal, and then, by relative inclination of the mirrors, to produce a system of straight fringes.

Owing to the dispersion introduced by prism P two such sets of fringes are formed corresponding to the two radiations. These sets are parallel for one condition of adjustment only—when relative inclination of the mirrors is about an axis parallel to the refracting edge of the prism; a photographic record of the fringe system then formed is shown in figure 2. By suitable inclination of one of the mirrors the fringe systems can be arranged in any orientation; thus in figure 3 the systems are shown inclined to a horizontal line at angles of 39° and 143° respectively, in figure 4 at 12° and 170°, in figure 5 at 19° and 46° and in figure 6 at 23° and 154°. For comparison, in figure 7 is shown a system of parallel fringes obtained with the single radiation 5461 A.

It is seen that the intersecting points of the two fringe systems lie on straight lines, whatever the orientation, and that a series of grey bands appear which are always vertical (that is, parallel to the refracting edge of the prism). These bands are the loci of those points where a maximum of one fringe system coincides with a minimum of the other system; they are shown most prominently in figure 2.

If the prism is of poor quality, due either to imperfection of its surfaces or to inhomogeneity of the glass, both fringe systems are curved considerably; the grey bands remain perfectly straight and parallel, however, as shown in figure 8. A piece of window glass interposed between the prism and mirror produces the same effect as an inferior prism.

The characteristics of these grey bands are: (i) the bands are independent of both the inclination and the shape of the interference fringes, always remaining straight and sensibly parallel to the refracting edge of the prism; (ii) for the same prism and the same radiations the spacing between the bands is constant; (iii) the spacing between the bands decreases as the difference in wavelength of the two radiations increases; (iv) a parallel displacement of one of the interferometer mirrors produces a movement of the bands. A linear relationship exists between mirror displacement and wavelength difference of the radiations: this was established and discussed in a previous paper (Tsien 1945).

§ 3. EQUATIONS FOR THE GREY BANDS

Single radiation

For a single radiation of wavelength λ and a small angle of inclination i between the interferometer mirrors the spacing a between two adjacent fringes is given by

$$a=\lambda/2i. \qquad \ldots\ldots(1)$$

Orientation of the two fringe systems

Assume that the refracting edge of the prism is parallel to the z axis (figure 9), and that after refraction the normals to the wavefronts corresponding to the two neighbouring radiations, and designated by λ_1 and λ_2, lie in the xy plane with λ_1 directed along the x axis and λ_2 inclined to it at an angle $\Delta\theta$.

Let the direction cosines of the normal to mirror M_2 be $\cos\alpha$, $\cos\beta$ and $\cos\gamma$, where α is a small quantity and both β and γ are very nearly equal to $\pi/2$. Mirror M_1 is assumed perpendicular to the single incident beam incorporating λ_1 and λ_2.

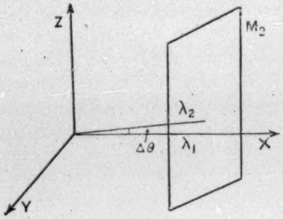

Figure 9.

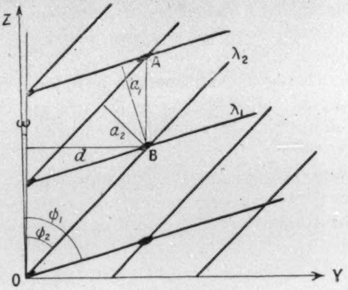

Figure 10.

First, consider the case of λ_1. If the virtual intersecting angle between M_1 and M_2 is δ_1, it is evident that $\delta_1 = \alpha$. The direction cosines of the intersecting line of the two mirrors will be

$$\cos\xi_1 = 0, \qquad \cos\eta_1 = -\cos\gamma/\sin\delta_1, \qquad \cos\zeta_1 = \cos\beta/\sin\delta_1.$$

The interference fringes of λ_1 will run in this direction.

Again, consider the case of λ_2. The virtual intersecting angle δ_2 between the two mirrors will have the relation

$$\cos\delta_2 = \cos\alpha\cos\Delta\theta + \cos\beta\sin\Delta\theta. \qquad \ldots\ldots(2)$$

The direction cosines of the intersecting line of the two mirrors will be

$$\cos\xi_2 = \sin\Delta\theta\cos\gamma/\sin\delta_2, \quad \cos\eta_2 = -\cos\Delta\theta\cos\gamma/\sin\delta_2,$$
$$\cos\zeta_2 = (\cos\Delta\theta\,\cos\beta - \sin\Delta\theta\cos\alpha)/\sin\delta_2.$$

The interference fringes of λ_2 will run in this direction.

Projection of the fringes of λ_2 into the yz plane

Since we observe the pattern of the interference fringes in the yz plane, we must project the fringes on to that plane.

Let ϕ_1 and ϕ_2 be the angles made with the z axis by the lines of the two systems of fringes of λ_1 and λ_2 when projected on the yz plane.

It is evident that for λ_1 we have $\phi_1 = \zeta_1$, so that $\sin\phi_1 = -\cos\gamma/\sin\delta_1$, $\cos\phi_1 = \cos\beta/\sin\delta_1$, and for λ_2 we have $\cos\eta_2 : \cos\zeta_2 = \sin\phi_2 : \cos\phi_2$, so that $\sin\phi_2 = -(\cos\Delta\theta\cos\gamma)/\epsilon$, $\cos\phi_2 = (\cos\Delta\theta\cos\beta - \sin\Delta\theta\cos\alpha)/\epsilon$, where

$$\begin{aligned}\epsilon^2 &= \sin^2\delta_2 - \sin^2\Delta\theta\cos^2\gamma \\ &= \cos^2\Delta\theta\sin^2\alpha + \sin^2\Delta\theta\cos^2\alpha - 2\sin\Delta\theta\cos\Delta\theta\cos\alpha\cos\beta.\end{aligned}$$

Since α, $\Delta\theta$, δ_1 and δ_2 are small quantities, the above relation becomes

$$\epsilon^2=\delta_2^{\,2}=\alpha^2+\Delta\theta^2-2\Delta\theta\cos\beta, \qquad \ldots\ldots(3)$$

to the first order of approximation.

Here we may note that $\phi_2=\zeta_2$ to the first order of approximation.

It is also useful to note the two following relations (to the first order of approximation),

$$\sin(\phi_2-\phi_1)=(\Delta\theta\cos\gamma)/\delta_1\delta_2, \quad \cos(\phi_2-\phi_1)=(\delta_1^{\,2}-\Delta\theta\cos\beta)/\delta_1\delta_2.$$

Intersection of the two systems of fringes

Suppose that the origin is one of the intersection points of the two systems of fringes. Since we consider all fringes in the yz plane, we obtain the equation for the first system of fringes of λ_1 as

$$(y-ma_1\cos\phi_1)/\sin\phi_1=(z+ma_1\sin\phi_1)/\cos\phi_1,$$

and that for the second system of fringes of λ_2 as

$$(y-na_2\cos\phi_2)/\sin\phi_2=(z+na_2\sin\phi_2)/\cos\phi_2,$$

where m and n are positive and negative integers, including zero, and a_1 and a_2 are the respective fringe separations of the two systems.

For the point of intersection we find

$$y=(na_2\sin\phi_1-ma_1\sin\phi_2)/\sin(\phi_1-\phi_2),\; z=(na_2\cos\phi_1-ma_1\cos\phi_2)/\sin(\phi_1-\phi_2).$$

By putting $m=n+k$ and letting n vary while keeping k as a constant integer, we find the equation for the system of lines formed by the intersection of these infinite number of points by eliminating n from the above two expressions, obtaining

$$\frac{y+ka_1\sin\phi_2/\sin(\phi_1-\phi_2)}{a_2\sin\phi_1-a_1\sin\phi_2}=\frac{z+ka_1\cos\phi_2/\sin(\phi_1-\phi_2)}{a_2\cos\phi_1-a_1\cos\phi_2}. \qquad \ldots\ldots(4)$$

This represents a system of parallel lines as k varies integrally, corresponding to the loci of the points of intersection of fringes of the two systems, maxima coinciding to give rows of points of high illumination (seen as finite areas), minima coinciding to give rows of dark points, and maxima of one system coinciding with minima of the other to give the grey bands.

Verticality of the bands

Suppose that the bands are inclined at an angle ω to the z axis which is the direction of the refracting edge of the prism P, then

$$\sin\omega=\frac{a_2\sin\phi_1-a_1\sin\phi_2}{\sqrt{\{a_1^{\,2}+a_2^{\,2}-2a_1a_2\cos(\phi_1-\phi_2)\}}}. \qquad \ldots\ldots(5)$$

Since by (1) $a_1=\lambda_1/2\delta_1$ and $a_2=\lambda_2/2\delta_2$, we obtain, using the relation of equation (3),

$$\sin\omega\simeq\frac{(\lambda_2-\lambda_1)\cos\gamma}{\sqrt{\{\lambda_1\lambda_2(\Delta\theta)^2+(\lambda_2-\lambda_1)(\lambda_2\delta_1^{\,2}-\lambda_1\delta_2^{\,2})\}}}\simeq\frac{(\lambda_2-\lambda_1)\cos\gamma}{\Delta\theta\sqrt{(\lambda_1\lambda_2)}}\simeq\frac{(\lambda_2-\lambda_1)\cos\gamma}{\lambda\Delta\theta}. \qquad \ldots\ldots(6)$$

Although $\cos\gamma$ can be varied at will, γ is always very near to $\pi/2$, so that ω varies very slightly and is practically equal to zero, for $\lambda_2-\lambda_1=\Delta\lambda$ is negligible compared with λ. When the two systems of fringes are vertical and parallel, $\cos\gamma=0$ and $\omega=0$ exactly. This is the only condition for the exact verticality of the bands. On the other hand, it is not difficult to see that the two systems of fringes cannot be horizontal at the same time.

For a rough estimate of the magnitude of ω, take the case of the mercury yellow doublet with a prism of light flint, and assume that γ deviates about 1′ from 90°. It is found that ω is about 10′.

Spacing of the bands

From geometry the relation for the spacing of the bands is

$$d=a_1a_2/\sqrt{\{a_1^2+a_2^2-2a_1a_2\cos(\phi_1-\phi_2)\}}.$$

By using this formula to the first order of approximation, and making the same substitutions as were used in deriving equation (6) from equation (5), we find

$$d\simeq\sqrt{(\lambda_1\lambda_2)}/2\Delta\theta\simeq\lambda/2\Delta\theta. \qquad \ldots\ldots(7)$$

It will be noted that the mathematical form of equation (7) is exactly the same as that of equation (1). Comparing the physical meaning of these two equations, we see that the spacing of the bands is equal to the spacing of one system of fringes when the other system has been made null. The above statement has been proved experimentally. After a few trials, no difficulty is found in adjusting mirror M_2 so that it is perpendicular to one radiation. Figure 12* is a photograph taken under these conditions. The fringes are not very straight owing to the imperfect quality of the prism. It is also obvious that under this condition the interference fringes must be parallel to the refracting edge of the prism, if the quality of the latter is fairly good. For when M_2 is perpendicular to λ_1, we have $\cos\alpha=1$, $\cos\beta=0$, $\cos\gamma=0$, and therefore $\sin\phi_2=0$.

§ 4. DISPERSION

It is apparent from equation (7) that the dispersion ($d\theta/d\lambda$) of the prism in use can be determined by measuring the spacing of the grey bands. Such a determination was made for various prisms, and the results obtained were in good agreement with those based on a calculation using Hartman's dispersion formula.

§ 5. THE RESOLUTION OF Hα

The cross-pattern of interference fringes may offer a new contribution to the resolution of spectral lines. The existence of bands formed by the intersection of the two systems of fringes serves as a good indication of the success in resolution. The spacing between two bands gives the dispersion of the prism for these radiations, and the shifting of the bands as one of the reflecting mirrors is displaced yields the value of the difference of the two radiations in question (Tsien 1945). For convenience in measurements, the spacing d between two bands is brought to a suitable width, from 1 mm. to a few centimetres. For a large difference between the wavelengths of two radiations, a prism of low dispersion glass and small refracting angle is chosen, and conversely, for a small difference between the wavelengths a prism of high dispersion glass and large refracting angle is preferable.

* Figure 11 omitted during amendment.

An attempt to resolve the red Balmer hydrogen line ($H\alpha = 6562{\cdot}82$ A.) was made. It has been known as a doublet with a separation of about 0·13 A. The source of light was a water-cooled hydrogen discharge tube of low pressure excited at 2000 v. A 65° prism made of extra dense flint and of 8 cm. side was used; owing to the large incident angle at the prism surface, however, the aperture of the prism is considerably reduced. A rough estimate of the dispersion $d\theta/d\lambda$ of the prism at this radiation gives about 3000 radians per centimetre; then, by calculation, the spacing of the bands in this arrangement is about 8·4 cm. Thus in the experiment two bands cannot be seen simultaneously. In spite of this, the shifting of a single band should be detectable, as one of the reflecting mirrors makes a parallel displacement. According to the equation suggested in the previous paper (Tsien 1945), the displacement required to shift a band by one spacing is about 1·6 cm. As a matter of fact, in this experiment the interference fringes disappeared as the mirror made a displacement of only 4 or 5 mm. In a paper by Buisson and Fabry (1912) it was pointed out that the two components of $H\alpha$ from a hydrogen discharge tube at room temperature are large enough to overlap each other; if the tube were put into liquid air, they would be perfectly separated. This is due to the Doppler effect which predominates in a light gas at ordinary temperature. As no liquid air was available, this method could not be explored.

In order to reduce the spacings of the bands and make the intersection of the two systems of fringes visible, it is necessary to increase the dispersion of the prism. There are three ways of doing this. Firstly, increasing the number of prisms employed is certainly beneficial; it is preferable to put a second prism in the other beam of light so as to make the absorption in the two beams equal. However, the use of prisms in succession reduces the effective aperture of the prism. Secondly, the dispersion is increased by increasing the refracting angle of the prism; the consequent reduction of aperture is, however, accompanied by a loss of light. A full discussion of this point is given in E. C. C. Baly's *Spectroscopy*, pp. 66–72. Generally speaking, a prism of refractive index below 1·7 and refracting angle not more than 65° is suitable. Thirdly, a glass of high refractive index usually possesses high dispersive power, $dn/d\lambda$. Wood (1934) has discovered the remarkable optical properties of nitro-dimethyl-aniline. It has an enormous dispersive power in the yellow and green regions, the term $dn/d\lambda$ near $H\alpha$ being about 6600 cm^{-1}. If a 60° prism of such material is used, the separation of bands for the $H\alpha$ doublet can be reduced to no more than 2 cm. Moreover, imperfections in the optical quality of the organic prism, whilst being a serious drawback to its use in a spectrograph, in no way affect the formation of the grey bands.

§6. DISTORTION OF THE FRINGES

Since λ_1 and λ_2 differ by a very small amount only, any local inhomogeneity in the prism introduces substantially the same change in optical path, and this produces the same fractional fringe shift. Thus at a point of intersection of any two fringes of the two systems, in a region affected by the prism defect, each fringe is displaced the same fractional part of the corresponding fringe separation. By simple geometry, the point of intersection must be displaced along the line of intersection points on which the undisplaced point lies. Therefore the lines of intersection, and thus the grey bands, are unaffected by prism defects as regards both homogeneity and surface flatness.

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A Method of Fitting the Andrade Creep Equation to Experimental Results

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ABSTRACT. A method is described whereby the constants in Andrade's formula, $l=l_0(1+\beta t^{\frac{1}{3}})e^{\kappa t}$, for the flow of metals under constant stress, can be rapidly deduced from experimental results by direct reading from a system employing sliding templates of calculated shape. The general equation to which the method is applicable is deduced.

§ 1. INTRODUCTION

It is now generally accepted (for example Cottrell and Aytekin 1947, Orowan 1947, Latin 1948) that Andrade's formula $l=l_0(1+\beta t^{\frac{1}{3}})e^{\kappa t}$ represents very closely the creep of metals under constant stress, and effects a separation of the β, or transient, flow, and the κ, or permanent, flow, where each has a distinct physical significance (Andrade 1910, 1914, Andrade and Chalmers 1932). The formula, however, is a somewhat troublesome one to fit to experimental values, and in the past the method employed has been either that of determining β for small values of t, where the κ-term is small, and hence calculating κ from the results for a larger value of t, with suitable subsequent adjustment, or one of the " algebraic " methods which depend on the selection of particular times in order that an equation involving only one of the constants may be formed (the use of three times separated by equal intervals, for instance, provides a quadratic in β). All the methods are lengthy, and particularly tedious if a large number of curves have to be fitted. In the following an account is given of a method of obtaining values of β, κ and l_0, from any creep curve, by direct graphical reading.

§ 2. THEORY OF THE METHOD

1. It is convenient to put $l_0=f_1$, $(1+\beta t^{\frac{1}{3}})=f_2$ and $e^{\kappa t}=f_3$, and to use the notation $(\ )_1$, $(\ )_2$, ... to represent values at t_1, t_2. . . . Consider the curve $(f_2 f_3)_1=l/l_0=L$, where κ and β are treated as the variables. We require to know what effect altering the value of L has on the shape of the curve.

Differentiating we obtain $-Lt_1e^{-\kappa t_1}(d\kappa/d\beta)=t_1^{\frac{1}{3}}$ or $d\kappa/d\beta=-t_1^{-\frac{2}{3}}/(1+\beta t_1^{\frac{1}{3}})$, and $\underset{\beta\to0}{\mathrm{Lt}}\,(d\kappa/d\beta)=-t_1^{-\frac{2}{3}}$.

As the slope of this κ–β curve for a constant t depends only on β, then altering the value of L merely shifts the curve in a direction parallel to the κ-axis, its shape remaining the same. It remains to find how the shift is related to changes in the value of L. For $\beta=0$, and for the two curves $(f_2f_3)_1=M$ and $(f_2f_3)_1=N$, we have $\kappa_M=t_1^{-1}\log M$ and $\kappa_N=t_1^{-1}\log N$. Hence we may obtain the curve $(f_2f_3)_1=N$ by displacing $(f_2f_3)_1=M$ a distance corresponding to $t_1^{-1}\log(M-N)$, or, in other words, if a vertical logarithmic scale is added for the $M, N \ldots.$ values, then equal shifts with respect to this scale correspond to equal changes in the value of the L term.

2. The curves $(f_2f_3)_1=M$ and $(f_2f_3)_2=N$, will provide, at their point of intersection, values of β and κ which will satisfy both curves, that is, fulfil the required conditions at both t_1 and t_2. It is clear that the curves $(f_1f_2f_3)_1=M'$ and $(f_1f_2f_3)_2=N'$ are also of the same general form, the effect of change in the f_1 term being to displace the curve in a direction parallel to the κ-axis, just as change in the value of the M' term does. We require next to know how this point of intersection varies with change in the f_1 value. This may be obtained from

$$\frac{M'}{N'}=\frac{(1+\beta t_1^{1/3})}{(1+\beta t_2^{1/3})}\cdot e^{(t_1-t_2)\kappa}$$

for, differentiating,

$$\frac{M'}{N'}(t_2-t_1)e^{(t_2-t_1)\kappa}\frac{d\kappa}{d\beta}=(t_1^{1/3}-t_2^{1/3})(1+\beta t_2^{1/3})^{-2}$$

which reduces to $(1+\beta t_1^{1/3})(1+\beta t_2^{1/3})(t_2-t_1)(d\kappa/d\beta)=(t_1^{1/3}-t_2^{1/3})$. When $\beta=0$, $d\kappa/d\beta=(t_1^{1/3}-t_2^{1/3})/(t_2-t_1)=C$, a constant, for all values of f_1, once the times t_1 and t_2 have been chosen. At any other point, $d\kappa/d\beta=C$ multiplied by a function of β only. Therefore, as f_1 varies, the locus of the point of intersection of the curves $(f_1f_2f_3)_1=M'$ and $(f_1f_2f_3)_2=N'$ is of a determined shape, independent of the values of M' and N', but displaced parallel to the κ-axis an amount determined by the choice of their values (see figure 1). If a similar locus is obtained for times t_2 and t_3 (the time t_3 corresponding to a length O'), it is apparent that the point of intersection of the two loci represents a point of common values for β, κ and l_0 for the three curves

$$(f_1f_2f_3)_1=M',\ (f_1f_2f_3)_2=N'$$

and

$$(f_1f_2f_3)_3=O'.$$

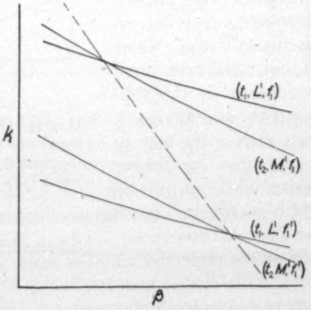

Figure 1.
— — — Locus of point of intersection of t_1 and t_2 curves as f_1 varies.

§ 3. METHOD

Using the foregoing theory, we proceed to a suitable graphical method for finding the values of the constants β, κ and l_0 from experimental curves. Graphs are plotted, on a suitably large scale, of κ against β for the three particular times selected : it has been shown that the *shape* of any one of these is fixed once t is fixed. From these curves, templates are cut from some suitable material, such as Bristol board. The method, then, depends upon finding some point at which all three curves will intersect (hence providing the same values for κ and β) and at which each template will indicate the same l_0. The next step in the construction is to mark each template with a logarithmic scale (AA′ in figure 2). This is best carried out by setting the extreme left of the template, i.e. the point X (figure 2) against those calculated values of κ which give $(f_2 f_3) = 1{\cdot}01$, $1{\cdot}02$, $1{\cdot}03$, ... $1{\cdot}1$ . . . . $1{\cdot}2$ . . . . etc., and inscribing these values on the template in relation to some fiducial mark Q. The three fiducial marks (one for each template) can, of course, be placed anywhere, but if these are as widely separated as possible, then the subsequent laying-out of the l_0-scale (described later) is made easier. If we now assign to these three fiducial marks a suitable round number for l_0, corresponding approximately to the original length of the experimental specimens (the value 20 cm. will be used throughout in the subsequent explanation), then each mark may be spread into logarithmic scales the points on which correspond to various values of l_0. These scales are easily laid out, the distance between say, 20·0 and 21·0 cm. being the distance between 1·00 and 1·05 on the respective template scale, and are preferably marked on a raised strip fixed to the card or board on which the diagram is drawn, so that the templates may be conveniently slid against this in a direction parallel to the κ-axis.

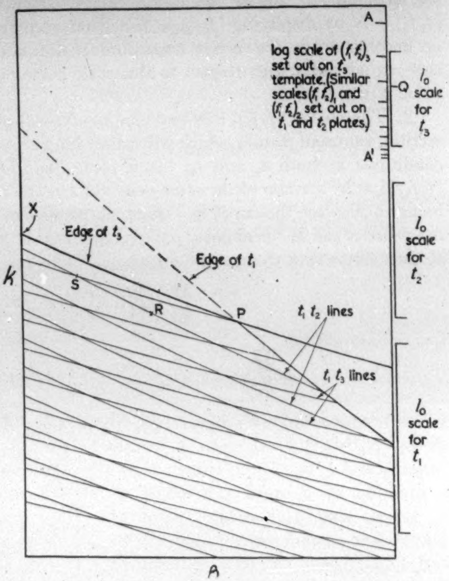
A
log scale of $(f_1 f_2)_3$ set out on t_3 template. (Similar scales $(f_1 f_2)_1$ and $(f_1 f_2)_2$ set out on t_1 and t_2 plates.)
Q
l_0 scale for t_3
A′
X
Edge of t_3
Edge of t_1
k
S
R
P
t_1 t_2 lines
t_1 t_3 lines
l_0 scale for t_2
l_0 scale for t_1
A

Figure 2.

Diagram shows the t_1 and t_3 templates in position providing the point P. If the setting of the t_1 and t_2 templates provides the point R then the point S is identifiable as that point for which values l_0, κ and β for the times t_1, t_2 and t_3 will coincide.

Values of l at times t_1, t_2 and t_3 are read off from the experimental curve and divided by 20 to give $(f_2 f_3)_1$, $(f_2 f_3)_2$ and $(f_2 f_3)_3$. Any l_0 value is chosen (20cm. for instance) and the templates are then so placed that the respective $(f_2 f_3)$ values are each set against this value of l_0. The curves will, in general, intersect in a small triangle, which may, by variation of the value of l_0 chosen, be reduced to a point, or at least to a minimum area. This would provide the answer we require, but by the use of the second part of the theory, if a permanent curvilinear grid is laid out on the graph, one set of lines corresponding to convenient displaced positions of the $t_1 t_2$ locus and the other set to similar positions of the $t_2 t_3$ locus, then the point required is identifiable by eye as the point of intersection of the two grid lines that pass through the points given by the intersection of the t_1 and t_2, and the t_2 and t_3 templates.

The " grid " may be obtained by taking the t_1 and t_2 templates and marking on the graph their point of intersection when each is set, with reference to its corresponding l_0 scale, against the same value of l_0. Points are plotted for various values of l_0 ,and the curve obtained gives the shape of all such loci for these two times. Similar curves are drawn by displacing this curve through suitable regular intervals in a direction parallel to the κ-axis. The procedure is repeated for two other templates, say t_2 and t_3, and the required grid is thus obtained. Figure 2 shows how the point required, S, is identified, and this point may now be " tested " for coincidence of all three templates ; the experimental $(f_2 f_3)$ values should now indicate the same l_0 on their respective scales, and the value of l_0 may therefore be read off from any one of them.

The grid lines themselves, of course, require no scaling, and by a judicious choice of the times t_1, t_2 and t_3 their slopes can be arranged to give an open and workable lattice.

There are many ways in which the use of a " grid " to locate the point required may be applied. It may, for instance, be laid out on a separate sheet, the two points provided by the initial settings of the plates being plotted on this grid and the common point identified as before. The most satisfactory method so far devised is that which uses a transparent sheet, inscribed, as in figure 3, with two lines from one "family" and one from the other (it will be evident that the point for the times t_1 and t_2 might be above or below, to the left or to the right of that for the times t_2 and t_3), the plate being moved laterally and vertically, that is, without rotation, into such a position that the $t_1 t_2$ and $t_2 t_3$ points lie on their respective loci. This is shown in figure 3.

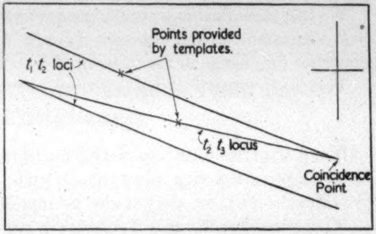

Figure 3.

Whichever method is used, the point obtained is checked by the templates and the value of l_0 read off. The value obtained may, of course, be either greater or less than that of the original setting.

§ 4. THE GENERAL EQUATION TO WHICH THE METHOD IS APPLICABLE

Consider the line given by the intersection of the surfaces

$$f(x, y, z, a) = A\ ;\quad f(x, y, z, b) = B. \qquad \ldots\ldots(1)$$

(The variables x, y and z in these generalized equations are equivalent to l_0, β and κ in the equation considered in the previous sections, a and b being equivalent to values of time, and A and B equivalent to values of length obtained experimentally.)

If the method discussed is to be applicable to this function, then firstly the projection of the line of intersection of two such functions on to the xy plane must be a curve whose shape remains unchanged by change in the value of A or B. This curve in the xy plane is equivalent to the locus of the point of intersection of two templates as l_0 varies. Secondly, the line of intersection of $f(x, y, z, a) = A$

with a plane $z=k$, a constant, must also be of such a form that its shape is unchanged by change in the value of A. This curve corresponds to the outline of the template itself.

Eliminating z from equation (1) we obtain the cylinder generated by lines parallel to the z-axis which pass through the line of intersection of the surfaces. The projection of this line on to the xy plane is given by the intersection of the cylinder and $z=0$, that is,

$$\phi(x, y, a, b)=\chi \text{ and } z=0. \qquad \ldots\ldots(2)$$

If this curve is to have the same shape for all values of χ, a change of the χ-value merely translating it along the y-axis, then it must be of the general form $y=F(x)+\chi$ where χ is an independent variable. Or, more generally, the required family may be written

$$\omega\{y-F(x)\}=\text{constant} \qquad \ldots\ldots(3)$$

where the constant has any assigned value. (The function $\omega(t)=\text{constant}$ also has the solution $\omega(t)=t^n$ as $n\to 0$. This, whilst it fulfils the condition, is not one to which the method may be applied, as the change in the values of the parameters with change in the values of the applied conditions becomes too small to be investigated by such a means.)

We may now deal in a similar manner with the second condition, that putting $z=k$, a constant, in the equation $f(x, y, z, a)=A$ provides a curve with similar properties for change in the value of A.

This is obviously fulfilled (as before) by an equation of the form

$$\delta\{y-\mathscr{F}(x)\}=\text{constant}. \qquad \ldots\ldots(4)$$

If a curve of this form is to result from putting $z=\text{constant}$ in either of equations (1), then the f-function must be of such a form that it may be written as $\delta\{y-\mathscr{F}(x)\}=\eta(z)$, for only if the z-function is separable does the function of $y-\mathscr{F}(x)$ remain unchanged for change in z, providing, for all functions and values of z, a family of curves with the required property.

Applying this, we may therefore write (1) in full as

$$\lambda\{y-F(x, a)\}\rho(z, a)=A\ ;\quad \lambda\{y-F(x, b)\}\rho(z, b)=B \qquad \ldots\ldots(5)$$

(retaining a, b, A and B in the notation).

If, now, our other condition is to apply, that is, that embodied in (3) the elimination of z between these two equations must provide a curve which may be written $\omega\{y-F(x)\}=\text{constant}$.

But the elimination of z from (5) provides, in general,

$$\Omega\{y-F(x, a), y-F(x, b), a, b\}=\text{constant}$$

(assuming we can obtain z on one side of at least one of the equations). If this is to be expressible in the form of (3), then either $F(x, a)=F(x, b)$ giving $a=b$, which is experimentally impossible, or, the elimination of z provides an equation of the form

$$\xi\{y-f_1(x)\}=\text{constant}\times\xi\{y-f_2(x)\} \qquad \ldots\ldots(6)$$

which can always be reduced to the form of (3).

The function of z, $\rho(z, a)$, must therefore be separable as $\mu(z)\psi(a)$, so that for all functions and values of these the curve given by (5) is reducible to that of (6) and hence to that of (3).

The general equation of the surface to which the method applies may therefore be fully stated as

$$\mathscr{F}_1\{y-f(x,a)\}\mathscr{F}_2(z)\mathscr{F}_3(a)=A. \qquad \ldots\ldots(7)$$

It follows, therefore, that if there are two quantities (a and A) between which an experimental relation has been formed, and that this relation can be expressed by an equation involving three constants x, y and z, then the method described can be used to find the values of the constants from any three readings on condition that (i) one of the constants is separable from the others and also from the experimental quantities, and (ii) that the remaining constants may be expressed in the form $\omega\{y-f(x)\}$.

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The Influence of a Transverse Magnetic Field on a Cylindrical Plasma

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ABSTRACT. The conditions in a cylindrical plasma under the influence of a transverse homogeneous magnetic field are treated by known methods. It is shown that the magnetic field deflects the column towards the wall with the result that the total loss of electrons and ions is increased. This causes an increase in electron temperature and in the axial field strength. Formulae relating the axial electric field, the magnetic field and the electron temperature are derived as well as equations describing the electron density distribution. Some measurements have been made in order to test the theory. The agreement is fair, taking account of the limits of experimental accuracy and of uncertainty in the constants used in the calculations.

§ 1. INTRODUCTION

The conditions in a cylindrical plasma under the influence of a transverse homogeneous magnetic field are treated in a manner analogous to that given by von Engel and Steenbeck (1934, pp. 82–87) for the case without magnetic field. The expressions for the electron drift in a magnetic field used here have been given by Tonks and Allis (1937).

The case to be treated is that in which the plasma is limited to a cylindrical tube and the current flowing in the direction of the axis. When a homogeneous magnetic field is applied at right angles to the axis the current is forced towards the tube wall thereby increasing the loss of electrons and ions. In order to compensate this loss the axial electron field increases, thus increasing the ionization and the electron temperature.

§ 2. THEORY

In the steady state the conditions in the plasma can be expressed by the equation

$$\operatorname{div}(Nv) = z'N, \qquad \ldots\ldots(1)$$

where N denotes the electronic or ionic density, v the drift velocity of the electrons and ions, and z' the average number of ion pairs created by one electron per unit of volume and time. Equation (1) will be valid under the assumptions that the electronic and ionic densities are approximately equal and that the recombination of ions and electrons and the formation of negative ions can be neglected.

A rectangular coordinate system is assumed with the z axis along the axis of the cylinder with its positive direction in the direction of the current. The magnetic field is in the y direction. Expression for the electron drift velocity components are given by Tonks and Allis (1937, equations (10) and (15)), viz.

$$\left.\begin{aligned} v_{x_e} &= -\alpha\left(b_e E_x + \frac{D_e}{N}\frac{\partial N}{\partial x}\right) - \beta b_e E_z, \\ v_{y_e} &= -b_e E_y - \frac{D_e}{N}\frac{\partial N}{\partial y}, \qquad v_{z_e} = -\alpha b_e E_z + \beta b_e E_x. \end{aligned}\right\} \ldots\ldots(2)$$

Here E_x, E_y and E_z are the components of the electric field, b_e and D_e are coeffiicents of mobility and diffusion, N is the electron density and α and β are dimensionless quantities defined by Tonks and Allis:

$$\left.\begin{aligned} \alpha &= 1 - h^2 + h^4 \exp h^2 \int_h^\infty \frac{\exp - h}{h}\, dh, \\ \beta &= \tfrac{1}{2} h \sqrt{\pi}\,(1 - 2h^2 + 4h^3 \exp h^2 \int_h^\infty \exp - h^2\, dh), \\ h &= eB\lambda/\boldsymbol{m}w, \end{aligned}\right\} \ldots\ldots(2\,a)$$

where B is the magnetic field, λ the electron mean free path and w is the most probable electron speed which is given in terms of the electron temperature T by

$$w^2 = 2\boldsymbol{k}T/\boldsymbol{m}.$$

The electron velocity distribution is supposed to be Maxwellian.

The corresponding expressions for the ionic velocity components are

$$v_{x_i} = b_i E_x - \frac{D_i}{N}\frac{\partial N}{\partial x} - b_i^2 B E_z,$$

$$v_{y_i} = b_i E_y - \frac{D_i}{N}\frac{\partial N}{\partial y}, \qquad v_{z_i} = b_i E_z,$$

because in this case h is very small and so $\alpha = 1$ and $\beta = b_i B$. For v_{z_i} the term $\beta b_i E_x$ is much smaller than $b_i E_z$ and so will be neglected.

In the steady state $v_{x_e} = v_{x_i} = v_x$ and $v_{y_e} = v_{y_i} = v_y$; we can eliminate E_x and E_y and the following expressions are obtained:

$$\left.\begin{aligned} v_x &= -\frac{1}{N}\frac{\partial N}{\partial x}\frac{b_i D_e + b_e D_i}{b_i + \alpha b_e}\alpha - b_e b_i E_z \frac{\beta + \alpha b_i B}{b_i + \alpha_e}, \\ v_y &= -\frac{1}{N}\frac{\partial N}{\partial y}\frac{b_i D_e + b_e D_i}{b_i + b_e}. \end{aligned}\right\} \ldots\ldots(3)$$

Since $\alpha b_e \gg b_i$ and

$$\frac{b_i D_e + b_e D_i}{b_i + b_e} \simeq \frac{D_e}{b_e} b_i = \frac{\boldsymbol{k}T}{\boldsymbol{e}} b_i = D_a \qquad \ldots\ldots(4)$$

the coefficient of ambipolar diffusion, we obtain

$$v_x = -\frac{D_a}{N}\frac{\partial N}{\partial x} - c, \qquad v_y = -\frac{D_a}{N}\frac{\partial N}{\partial y},$$

where

$$c = b_i E_z b_e \frac{\beta + \alpha b_i B}{b_i + \alpha b_e} \simeq b_i E_z \frac{\beta}{\alpha}. \qquad \ldots\ldots(5)$$

Equation (1) can now be written

$$D_a\left(\frac{\partial^2 N}{\partial x^2} + \frac{\partial^2 N}{\partial y^2}\right) + c\frac{\partial N}{\partial x} + z'N = 0.$$

After substituting $N = N' \exp(-cx/2D_a)$ and passing to cylindrical coordinates r and ϕ, the solution is obtained

$$N = N_0 \exp(-cr\cos\phi/2D_a) J_0\left\{r\sqrt{\left(\frac{z'}{D_a} - \frac{c^2}{4D_a^2}\right)}\right\}, \qquad \ldots\ldots(6)$$

where N_0 is the density along the z axis. Let $N = 0$ approximately at the wall, thus

$$r_0\sqrt{\left(\frac{z'}{D_a} - \frac{c^2}{4D_a^2}\right)} = 2{\cdot}405\ldots = n. \qquad \ldots\ldots(7)$$

r_0 is the tube radius and n the first zero of the Bessel function J_0. Equation (6) can be rewritten

$$N = N_0 \exp(-cr\cos\phi/2D_a) J_0(nr/r_0).$$

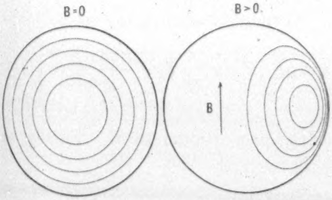

Figure 1.

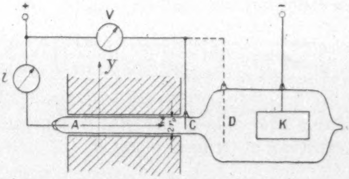

Figure 2.

This equation describes the electron density distribution. This is shown in figure 1 where the lines representing $N = \text{constant}$ are drawn in the cross section of the tube.

Now it is possible to express E_z and B as functions of the electron temperature T. According to von Engel and Steenbeck (1932, pp. 88–89) the rate of ionization of an electron is

$$z' = \frac{2amp}{\boldsymbol{e}\sqrt{\pi}}\left(\frac{2\boldsymbol{k}T}{\boldsymbol{m}}\right)^{3/2} \exp(-\boldsymbol{e}V_i/\boldsymbol{k}T)\left(1 + \tfrac{1}{2}\frac{\boldsymbol{e}V_i}{\boldsymbol{k}T}\right), \qquad \ldots\ldots(8)$$

where V_i is the ionization potential and p the pressure in mm. Hg. Equation (8) is the same as their equation (115) except for the factor 2 (instead of 600) as we are using here M.K.S. instead of C.G.S. units. Equation (8) is derived under the simplifying assumption that the electron mean free path is independent of the electron temperature. Values of the constant a are also given by von Engel and Steenbeck.

From (2) and (3) we obtain the axial component of the electron velocity

$$v_{z_\mathrm{e}} = -b_\mathrm{e}E_z\left(\alpha + \frac{\beta^2}{\alpha} - \frac{\beta^2 b_\mathrm{i}/\alpha + \beta b_\mathrm{i}^2 B}{b_\mathrm{i} + \alpha b_\mathrm{e}}\right) + \frac{\beta}{N}\frac{\partial N}{\partial x}\left(\frac{D_\mathrm{a}}{\alpha} - D_\mathrm{e}\right).$$

Here $\beta^2 b_\mathrm{i}/\alpha + \beta b_\mathrm{i}^2 B = (1 + \alpha b_\mathrm{i} B/\beta)\beta^2 b_\mathrm{i}/\alpha$ and it is obvious that $\alpha b_\mathrm{i} B/\beta \ll 1$ since for very strong magnetic fields α/β approaches zero. The last term in v_{z_e} is approximately equal to

$$-\beta\frac{D_\mathrm{e}}{N}\frac{\partial N}{\partial x} = -\frac{\beta}{N}b_\mathrm{e}\frac{\boldsymbol{k}T}{\boldsymbol{e}}\frac{\partial N}{\partial x}$$

and it is seen that this term can be neglected if $E_z \gg (\beta\boldsymbol{k}T/N\boldsymbol{e})\partial N/\partial x$. Thus by assuming as before $\alpha b_\mathrm{e} \gg b_\mathrm{i}$, we obtain $v_{z_\mathrm{e}} = -b_\mathrm{e}E_z(\alpha + \beta^2/\alpha)$, which means that in the presence of a transverse magnetic field the drift velocity is decreased by a factor $(\alpha + \beta^2/\alpha)$, the numerical value of which is 1·0 for zero magnetic field and then decreases when the magnetic field increases towards the value 0·883.

The electric field as a function of the electron temperature at zero field is, according to von Engel and Steenbeck (1932, pp. 183–185), but with regard to the above expression for the axial velocity,

$$E_z\left(\alpha + \frac{\beta^2}{\alpha}\right)^{1/2} = \frac{2\sqrt{2}}{\pi^{\frac{1}{2}}}\frac{\sqrt{\kappa}}{p\lambda}\frac{\boldsymbol{k}T}{\boldsymbol{e}}p. \qquad \ldots\ldots(9)$$

Here κ is the electron's average relative loss of energy by an impact with a molecule. The electron temperature T is very strongly affected by the magnetic field.

The equations (4), (5), (7)–(9) form a system from which the relation between E_z and B can be calculated. For the sake of simplicity we introduce a new variable u defined by the equation $u = \sqrt{(\boldsymbol{e}V_\mathrm{i}/\boldsymbol{k}T)}$. Then we obtain

$$\frac{\beta^2}{\alpha(\alpha^2+\beta)^2} = f(h) = \frac{1}{\kappa}\left[2\sqrt{\left(\frac{2\boldsymbol{e}V_\mathrm{i}}{\boldsymbol{m}}\right)}\frac{a(p\lambda)^2}{(pb_\mathrm{i})}u\exp(-u^2)(u^{-2}+\tfrac{1}{2}) - \frac{2{\cdot}88\pi(p\lambda)^2}{(pr_0)^2}\right]. \qquad \ldots\ldots(10)$$

From $(2a)$ follows that $f(h)$ is approximately proportional to h^2. Finally we express h in terms of u and B

$$h = -\lambda u B\sqrt{(\boldsymbol{e}/2\boldsymbol{m}V_\mathrm{i})}. \qquad \ldots\ldots(11)$$

The right-hand side of (10) is a function of u since κ is a function of u. For a certain value of u we can calculate $f(h)$ and then h. Then equation (11) gives the corresponding value of B and equation (9) that of E_z.

§ 3. EXPERIMENTS

Some measurements have been made on cold cathode glow discharges in different gases. Figure 2 shows the experimental arrangements.

The tubes were provided with a hollow cathode K, an anode A and a third electrode placed in one of the two positions C or D. According to this position the tubes will be divided into two groups referred to as type C and D. The tube was placed between the pole-pieces of an electromagnet. The distance between the anode and the third electrode is d and the length of the pole-pieces l.

Gas	p (mm. Hg)	Tube type	r_0 (cm.)	d (cm.)	l (cm.)	Anode current (ma.)	$E_{z_0}=\frac{V_0}{d}$ (v/cm.)	Calculated values of		Fig. No.
								E_{z_0} (v/cm.)	T_0 °K.	
H_2	5·0	C	0·3	3·5	3·5	15	71·5	74·5	18 400	3
H_2	2·7	D	1·0	6·0	4·5	10	33·4	34·8	16 400	3
N_2	0·43	D	1·0	6·0	4·5	7–12	12·5	—	20 300	4
N_2	2·5	D	1·0	6·0	4·5	7	45·0	24·6	14 500	4
He	0·9	C	0·25	3·5	3·5	20	24·3	—	61 800	5
He	1·4	C	0·25	3·5	3·5	5–20	21·7	—	51 500	5
He	2·8	C	0·25	3·5	3·5	5–20	24·6	—	41 000	5
Ne	1·1	C	0·3	3·5	3·5	10	12·9	13·3	51 400	6
A	0·57	D	1·0	6·0	4·5	5–10	10·5	—	17 400	6
A	1·8	D	1·0	6·0	4·5	5	15·5	—	14 200	6

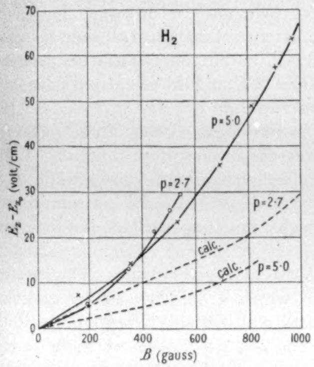

Figure 3. Increase in axial electric field as a function of magnetic field for hydrogen at different pressures and tube radii.

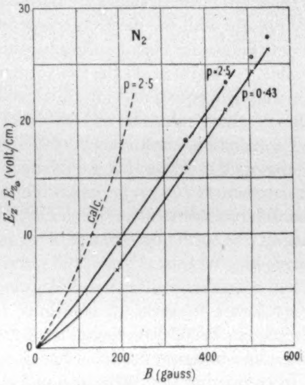

Figure 4. Increase in axial electric field as a function of magnetic field for nitrogen at different pressures.

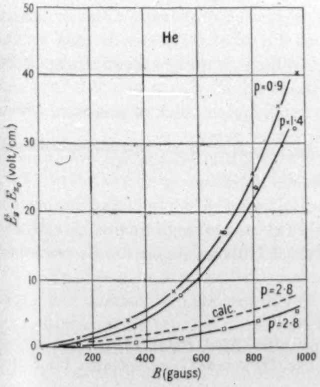

Figure 5. Increase in axial electric field as a function of magnetic field for helium at different pressures.

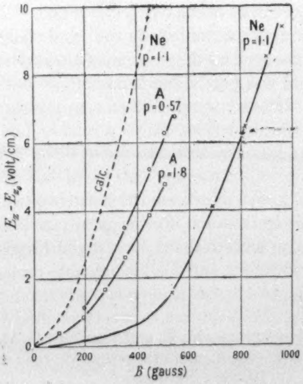

Figure 6. Increase in axial electric field as a function of magnetic field for neon and argon.

Full lines are experimental and broken lines are calculated from equations (9) and (10).

The voltage V between A and C (D) was measured with an ordinary voltmeter the resistance of which was $0{\cdot}5\times10^6$ ohms. Thus the current through the instrument was less than 1 ma. and the probe voltage probably accurate to within a few volts. The electric field E_z is then approximately $E_z-E_{z_0}=(V-V_0)/l$, where index zero refers to the magnetic field $B=0$. This method is not accurate but the measurements were made for the purpose of obtaining a first order check of the theory.

The tube data are given in the table and the relations between E_z and B are plotted in figures 3–6. The broken lines show the calculated relations. The values of κ used in the calculations are taken from von Engel and Steenbeck (1932, p. 187); these values are derived from Townsend's observations. κ varies very rapidly with the electron temperature, particularly in nitrogen; for helium κ is only known for elastic collisions, the occurrence of inelastic collisions increases its value. The value of κ has been obtained at three different electron temperatures by inserting in equation (9) the observed values of E_z and the calculated values of T at $B=0$; these values lie approximately on a straight line and give $\kappa=3{\cdot}4\times10^{-3}$ at $T=40\,000^\circ$ K. and $\kappa=9{\cdot}3\times10^{-3}$ at $T=60\,000^\circ$ K. Using these values in calculating the curves for $p=2{\cdot}8$ mm. gives the broken line in figure 5. The calculated curve for $p=1{\cdot}4$ mm. differs very little from that for $p=2{\cdot}8$ mm. The great difference between observed and calculated curves for the lower pressure is possibly due to the occurrence of strong high frequency oscillations which were observed.

For neon Druyvesteyn and Penning (1940) have given values of the electron energy losses in inelastic collisions from which κ can be calculated. Thus $\kappa=0{\cdot}058$ at $T=60\,000^\circ$ K. and $\kappa=0{\cdot}092$ at $T=80\,000^\circ$ K. Thus we obtain the broken line in figure 6.

In comparing the theoretical and the experimental results one must consider not only the simplifying assumptions made in the calculations and the uncertainty of the constants (especially κ) but also some disturbing effects in the measurements such as impurities in the gases. Only for the inert gases are the impurities supposed to have had negligible effects. The positive column was uniformly deflected towards the wall along the whole length of the tube. The anode glow was also influenced by the magnetic field and it is possible that the observed change in axial field was partly due to variations in the anode fall.

The measurements on nitrogen show a much higher value of axial field strength than calculation; this is probably due to traces of oxygen.

ACKNOWLEDGMENTS

I wish to express my great gratitude to Professor H. Alfvén, who has proposed this work to me, for his great interest and help in carrying out this investigation. I also wish to thank Dr. A. von Engel, Oxford University, for kindly reading the manuscript and for his valuable remarks.

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Note on the Ionic Conductivity of Oxide-Coated Cathodes

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MS. received 22 *December* 1948 *and in amended form* 12 *April* 1948

ABSTRACT. Ionic conductivity in an oxide-coated cathode is regarded as due to the mobility of oxygen ions. The magnitude of the conductivity can be derived from the phenomenon of current decay in a pulsed diode. Results are consistent with those obtained by other methods. Some consideration is given to the relation between ionic conductance and diffusion, and it is shown that the decay phenomenon results from the interaction of these phenomena in a thin layer on the outer surface of the coating.

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§ 1. INTRODUCTION

Values of ionic conductivity of oxide-coated cathodes have recently been calculated by Sproull (1945) from decay times of pulsed emission currents. The coatings employed were of the mixed crystal type. Sproull assumed that the current decay occurred as a result of increased surface work function due to ionic conductance of barium ions away from the emitting surface. The values of ionic conductivity derived by Sproull, lying between 5% and 50% of the total conductivity, are rather high and do not correspond with values measured by other methods.

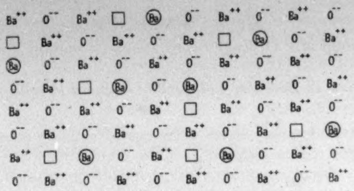
It is the object of this note to present an explanation of the decay phenomenon which differs materially from that of Sproull, and leads to results which are more consistent with known magnitudes of ionic conductivity.

§ 2. ORIGIN OF IONIC CONDUCTANCE IN THE COATING

If the decay effect results from a temporal increase in surface work function during the current pulse, this increase may be regarded as due either to a flow of positive barium ions away from the surface, or to a flow of negative oxygen ions towards the surface. The second assumption is considered by the writer as the more probable and it will therefore be examined in some detail.

The oxide-coated cathode is now regarded as a reduction semiconductor containing an excess of barium atoms. The manner in which these excess atoms are built into the crystal lattice of the earth alkali oxide has been examined by Schottky (1935) and by Jost and Nehlep (1936). From their investigations of the monovalent NaCl lattice they concluded that the lattice defects of this compound are due to formation of vacancies. Similar conclusions were reached for the bivalent earth alkali oxides and in particular for BaO.

For example, in a pure BaO cathode, the crystal lattice of the unactivated coating will contain a certain number of vacancies at oxygen ion sites and a roughly equal number at barium ion sites. By activation an excess of barium is obtained, and this results from an increase in the number of vacancies in the lattice of oxygen ions together with a decrease in the number of vacancies of barium ions. A well activated cathode of BaO will therefore contain a large number of vacancies in the oxygen lattice and only a small number in the barium lattice. If the latter number is neglected, a cross section through the BaO lattice will follow the configuration shown in the figure. The excess barium atoms, lying beside the vacancies, are indicated by circles in this figure.



BaO lattice in activated state.

The conductivities of the two kinds of ions depend on the number of vacancies belonging to each kind, and on their respective mobilities. Little knowledge exists of the relative magnitudes of these mobilities, but the number of vacancies appropriate to the oxygen ions has been shown to be much greater than the number appropriate to barium ions. There is a strong probability therefore that the part played by the oxygen ions in the total ionic conductivity is much greater than that of the barium ions. If, then, the decay phenomenon is to be explained by ionic conductivity, it is to the conductance of oxygen ions towards the surface rather than to the conductance of barium ions away from the surface that attention should be directed. Bearing this distinction in mind the problem can now be developed on lines similar to those employed by Sproull. To facilitate comparison with Sproull's paper, similar symbols have been used where possible and cross reference made to certain of Sproull's equations. The symbols are set out below.

N = number of oxygen ions per cm^2 of an atom layer of the BaO-crystal, N_c = number without vacancies, $N_{\min}$ = number for best activation, i = density of saturated emission current, $\mathbf{e}$ = electronic charge, t = time, κ_i = ionic conductivity, κ_e = electronic conductivity, D = diffusion constant, d = lattice spacing, Δ = thickness of layer in which ionic conductance and diffusion take place.

§ 3. CALCULATION OF THE DECAY OF EMISSION

When a current of density i traverses the coating from the surface to the core material, a number of oxygen ions are conducted to the atom layers near the surface. The flow of oxygen ions will reduce the number of oxygen vacancies near the surface, which in effect, results in an equal numerical reduction of excess barium there. The effect is analogous to the reduction of excess barium in the surface layer, such as occurs when oxygen ions impinge upon the surface from residual components of the valve atmosphere. In both cases the external work function increases, since this depends on the number of excess barium atoms in or near the surface. A decay of emission is therefore observed. The anomalous result of a decreased work function, observed by Sproull, may be due to the fact that he used the Richardson method which is not suitable for oxide-coated cathodes (Herrmann and Wagener 1944).

As $\kappa_e \gg \kappa_i$, the number of conducted ions N_i is given by

$$dN_i/dt = i\kappa_i/2\mathbf{e}\kappa_e. \qquad \ldots\ldots(1)$$

This flow of oxygen ions towards the surface sets up a concentration gradient through the coating and results in a diffusion flow of oxygen ions in the opposite direction. The number of these diffusing ions N_d is given by

$$dN_\mathrm{d}/dt = -(N-N_0)D/\Delta d, \qquad \ldots\ldots(2)$$

where the subscript 0 denotes quantities and currents at time $t=0$. The effective number of ions, moving to the surface, is the vector sum of equations (1) and (2). Thus

$$\frac{dN}{dt} = \frac{\kappa_\mathrm{i} i}{2e\kappa_\mathrm{e}} - \frac{D}{\Delta d}(N-N_0), \qquad \ldots\ldots(3)$$

which corresponds to equation (4) in Sproull's paper.

For evaluating equation (3) the dependence of emission current on the number of oxygen ions in the surface layer must be known. This dependence can be obtained from experiments of Becker (1929) who used the oxide coating as an anode and sent electrons, coming from a metal cathode outside, into the coating. This procedure gives exactly the opposite effect to that observed by Sproull. In Becker's experiments the ionic current, accompanying the electron current, flowed into the oxide coating from the surface towards the core. This resulted in a movement of oxygen ions from the surface into the interior and gave rise to the phenomenon of decreased external work function and increased emission. According to Becker the logarithm of the increase in emission is approximately a linear function of the number of oxygen ions removed. Let i_{N_c} and i_{N_min} denote the saturated emission currents appropriate to surface states with N_c and N_min oxygen ions respectively. Then, according to Becker,

$$\ln(i/i_{N_\mathrm{c}}) = b(N_\mathrm{c}-N) \quad \text{and} \quad \ln(i_{N_\mathrm{min}}/i_{N_\mathrm{c}}) \simeq b(N_\mathrm{c}-N_\mathrm{min}). \qquad \ldots\ldots(4)$$

Introducing the abbreviation

$$L \simeq \ln(i_{N_\mathrm{min}}/i_{N_\mathrm{c}}), \qquad \ldots\ldots(5)$$

the ratio of the two equations in (4) gives

$$\ln(i/i_{N_\mathrm{c}}) = L(N_\mathrm{c}-N)/(N_\mathrm{c}-N_\mathrm{min}). \qquad \ldots\ldots(6)$$

Equation (6) gives the required relation between emission current and density of oxygen ions in the atom layer at the surface. The equation differs from the corresponding equation (5) of Sproull in the value of the ratio on the right hand side. Sproull assumes the existence of a monatomic layer of barium on top of the coating and uses the ratio N/N_c (N denoting the density of barium instead of oxygen). The existence of such a monatomic layer, however, is very unlikely, if the composition of the activated oxide coating is as shown in the figure (cf. Herrmann and Wagener 1944).

If equation (6) is now rewritten for time $t=0$, and the result subtracted from equation (4), then

$$\ln(i/i_0) = L(N_0-N)/(N_\mathrm{c}-N_\mathrm{min}). \qquad \ldots\ldots(7)$$

Differentiating and combining with (3) gives

$$\frac{1}{i}\frac{di}{dt} = \frac{-L}{N_\mathrm{c}-N_\mathrm{min}}\frac{dN}{dt} = \frac{DL}{\Delta d}\,\frac{N-N_0}{N_\mathrm{c}-N_\mathrm{min}} - \frac{\kappa_\mathrm{i} L i}{2e\kappa_\mathrm{e}(N_\mathrm{c}-N_\mathrm{min})}. \qquad \ldots\ldots(8)$$

Combining (7) and (8) with

$$N_c - N_{\min} = pN_c \qquad \ldots\ldots(9)$$

results in the differential equation for the emission current

$$\frac{1}{i}\frac{di}{dt} = \frac{D}{\Delta d}\ln\left(\frac{i_0}{i}\right) - \frac{\kappa_i L}{2e\kappa_e p N_c}\,i. \qquad \ldots\ldots(10)$$

§4. ASCERTAINING THE IONIC CONDUCTIVITY

The only difference between this and the corresponding equation (9) of Sproull is the appearance of the factor p. Solution of the equation leads accordingly to the same result as that of Sproull changed only by a constant factor. This factor is, however, decisive for the calculated magnitude of ionic conductivity. Thus, if $\kappa_i^{(\mathrm{Ba})}$ is the conductivity calculated by Sproull for conductance by barium ions, and $\kappa_i^{(\mathrm{O})}$ is the conductivity for conductance by oxygen ions, then $\kappa_i^{(\mathrm{O})}/p = \kappa_i^{(\mathrm{Ba})}/1$ or $\kappa_i^{(\mathrm{O})} = p\kappa_i^{(\mathrm{Ba})}$.

It will be apparent from (9) that p is the relative difference between the number of oxygen ions in a complete atom layer and the corresponding number in a fully activated layer with a maximum number of vacancies. This difference is equal to the relative number of excess barium ions in the fully activated layer. According to most analyses the percentage of excess barium atoms in a fully activated cathode is of the order of 0·2% and therefore $p = 0{\cdot}002$ (Clausing 1935, Isensee 1937). Hence the ionic conductivity, calculated for the conductance of oxygen ions will be only 2×10^{-3} times as large as the value calculated for barium ions by Sproull. From Sproull's values of $\kappa_i^{(\mathrm{Ba})}$ between 5% and 50%, the derived values for $\kappa_i^{(\mathrm{O})}$ are therefore between 0·01% and 0·1%. These values correspond well with the value of 0·05% measured for BaO by Isensee (1937), but they are still somewhat higher than the value of 0·005% obtained for mixed crystals of BaO/SrO measured by Berdennikowa (1932) and 0·001% by Isensee. The earlier result of 0·5% by Becker (1929) is based on the assumption of a monatomic layer on top of the coating. Reconsideration of this result according to the concepts of surface described in this paper will give a value of the same order as those obtained by the other workers.

§5. APPLICATION OF THE RELATION BETWEEN IONIC CONDUCTIVITY AND DIFFUSION CONSTANT

Further consideration will now be given to the relation existing between ionic conductivity κ_i and diffusion constant D. So far this relationship has been neglected, but its use leads to the removal of a further difficulty arising from Sproull's treatment. According to Wagner (1931)

$$D = \boldsymbol{k}Tu_i/v\boldsymbol{e}, \qquad \ldots\ldots(11)$$

where v = valency, $\boldsymbol{k}$ = Boltzmann's constant, T = absolute temperature, u_i = ionic mobility. From the value of ionic mobility

$$u_i = \kappa_i/(v\boldsymbol{e}N_0/d), \qquad \ldots\ldots(12)$$

is derived the relationship

$$D = 4{\cdot}31 \times 10^{-5}Tu_i = 4{\cdot}31 \times 10^{-2}(\kappa_i d/2\boldsymbol{e}N_0), \qquad \ldots\ldots(13)$$

for a temperature of 1000° K.

Applying (10) for the final state of equilibrium in which $di/dt=0$ and denoting the current flow during this state by i_∞, an expression for D gives

$$D=\frac{\kappa_1 L\Delta d}{2e\kappa_e p N_c}\times\frac{i_\infty}{\ln(i_0/i_\infty)}. \qquad \ldots\ldots(14)$$

Using this value of D in (13) leads to an expression for the thickness of the layer of coating in which diffusion takes place. Thus

$$\Delta=4{\cdot}31\times10^{-2}\,\frac{\kappa_e p N_c}{L N_0 i_\infty}\ln(i_0/i_\infty). \qquad \ldots\ldots(15)$$

As $N_c/N_0\simeq1$, then for $\kappa_e=5\times10^{-3}$, $L=\ln(i_{N_{\min}}/i_{N_c})=10$, $i_\infty=0{\cdot}5$ amp. and $\ln(i_0/i_\infty)=2$, we find $\Delta=17\times10^{-8}$ cm. With a lattice spacing for BaO of $2{\cdot}76\times10^{-8}$ cm., the derived value of Δ corresponds to about 6 atom layers.

The result indicates that the ionic conductance and diffusion which cause the decay of emission only take place in a thin layer on the surface of the coating. This is consistent with the fact, observed by Sproull, that the decay phenomenon is independent of the thickness of the coating. The absolute value obtained for Δ will not be considered here on account of the inaccuracy of the experimental values used for the calculation of Δ.

For further consideration of the problem it will become necessary to take into account the loss of oxygen ions from the outer surface by evaporation. Equation (10) might be adapted to cover this evaporation.

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Emission of Secondary Electrons from Nickel and Molybdenum by Neutral Atoms of Mercury and Potassium

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Communicated by M. L. Oliphant ; *MS. received* 14 *June* 1947, *in amended form* 29 *April* 1948

ABSTRACT. Investigations have been carried out on the emission of secondary electrons from Ni and Mo produced by high speed neutral atoms of mercury and potassium. The mercury particles were produced by neutralizing a homogeneous beam of accelerated ions from a hot-cathode mercury arc on the walls of a narrow nickel canal which they strike at grazing incidence. The neutral atoms of potassium were produced by passing a beam of fast ions of definite velocities through vapours of potassium where they are neutralized by electron exchange with neutral K atoms. It is observed that the secondary emission rises with the number and the energy of the particles and that the energy distribution among the secondary electrons is always Maxwellian, suggesting that the emission is probably due to intense local heating. The effective temperatures of the target have been calculated in typical conditions.

It is also shown that the targets scatter neutral atoms, dirty surfaces scatter more Hg atoms than clean ones but the reverse is the case with K particles.

§ 1. INTRODUCTION

THE emission of secondary electrons from metal surfaces by positive ions, both slow and fast, is well established but very few data are available for the secondary emission by neutral atoms. Webb (1924), Messenger (1926), Coulliette (1928) and Sonkin (1933) have investigated the secondary emission by very slow metastable atoms of mercury. Oliphant (1930) studied the energy distributions among the secondary electrons emitted from Ni and Mo by fast metastable atoms of helium.

The present work is devoted to the study of the secondary emission from Ni and Mo by high energy neutral atoms of mercury and potassium.

§ 2. APPARATUS

The apparatus and electrical connections are shown in figure 1. The filament F_1, a thin tungsten spiral surrounded by a Ni spiral grid G, was held in position in the centre of a pyrex glass bulb A about 14 cm. in diameter. A Ni metal probe P, 4 cm. in diameter, with a circular hole 3 mm. wide at its centre and a Ni canal 5 cm. long and 3 mm. in diameter spot welded normal to its surface at the back, was sealed in a side tube in A opposite to F_1. The Ni disc S_1 was 1·8 cm. in diameter and had a hole 3·5 mm. in diameter at its centre. The Ni cylinder S_2, 4 cm. long and 1 cm. in diameter, had 4 mm. circular holes at the centre of its closed ends. The Ni cylinder C_2, 3·5 cm. long and 2·5 cm. in diameter, was open at one end with a hole 5·5 mm. in diameter at the centre of its closed end and served to collect the secondary electrons emitted from the metal target, T, which was a circular disc 1·8 cm. in diameter. The Ni cylinder C_1, 4·2 cm. long, 3 cm. in diameter, and open at one end had a circular hole 4·5 mm. in diameter at the centre of its other end and enclosed the cylinder C_2 to shield it from stray charges.

P_1 and P_2 were two rectangular parallel Ni plates each 2·5 cm. long and 1·5 cm. wide and were kept 5–6 mm. apart. The axes of the canal, the cylinders C_1, C_2, S_2 and the plates P_1 and P_2 were carefully adjusted to lie on the straight line passing through the centre of S_1. The target T was spot welded to a Ni tube C which, when desired, could be pulled back right out of the cylinder C_2 against the pressure of a tungsten spiral spring S. This is done by coiling the Ni flexible wire W round J by rotation of the ground-glass joint G_2. The tungsten filament F_2 served to degas the target by electron bombardment. The whole apparatus was made of pyrex glass and all the metal parts were made of Ni or tungsten.

The apparatus was exhausted to a pressure below 10^{-6} mm. Hg by a two-stage all-steel mercury diffusion pump backed by a Cenco Hyvac pump and the mercury vapours were checked from entering the system by condensing them in the liquid air trap M. The experimental mercury in H was pure metal and was twice distilled under vacuum before use. The apparatus was tested to be vacuum tight and was baked at 500° c. for four hours before any observations were taken. The pumps were kept continuously running during the experiments and the gas pressure inside was read on a McLeod gauge.

The voltage for accelerating the particles was obtained from a D.C. motor generator set giving 0–5500 volts at 15 ma. and was read on electrostatic voltmeters.

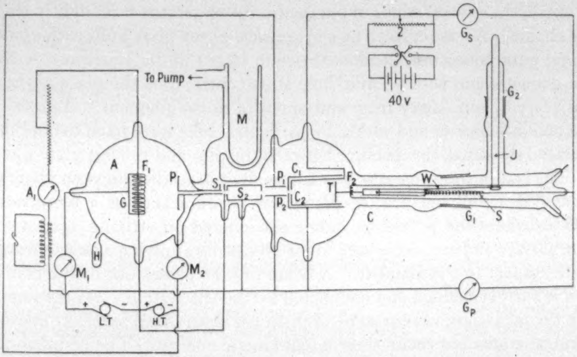

Figure 1.

The hot cathode mercury arc was maintained in A at 100–110 V. D.C. and the positive ions formed in the ionized gas were drawn to P by applying a negative potential to it with respect to the arc. Some of the positive ions arriving at P at normal incidence enter the canal and a fraction of them are neutralized by collision with its walls at grazing incidence without losing their energy, which is equal to the accelerating potential applied to the probe. The ions which pass right through the canal are stopped from reaching the target T and its surrounding system by a suitable reverse field applied between the canal and S_1 and S_2 and by the field between the parallel plates P_1 and P_2. The accelerated neutral particles emerging out of the canal are, however, not influenced by these fields, but reach the target T and liberate therefrom secondary electrons which are collected on C_2 and measured by the galvanometer G_s which has a current sensitivity of 10^{-10} amp/mm. Any positive ion current going to the target system is detected by the galvanometer G_p which is as sensitive as G_s.

With electrical connections on as shown in figure 1, but with no heating current in F_1, no deflection was observed in G_p or G_s; this indicates good electrical insulation of the whole system. On switching the mercury arc in A by heating the filament F_1 no current whatsoever was detected by G_p but an appreciable electron current flowing from the target T to the collector C_2 was registered by G_s. That the emission was from the target was verified by pulling it right out of the collector; the current then fell to zero but recovered to its full value as soon as the target was brought in position inside C_2. As no positive ion current was flowing to T or any of the cylinders C_1 or C_2 it was concluded that the observed secondary emission was either due to the radiations coming from the arc or was caused by the neutral atoms of mercury hitting the target or was due to both these effects. On switching off the positive ion accelerating voltage applied between F_1 and P, but keeping all other connections the same, the secondary current fell to zero, showing that the radiations from the arc were not causing this emission. It was therefore concluded that the neutral atoms of mercury formed in the canal and reaching the target were responsible for the observed secondary emission. The energy distribution among the secondary electrons was studied by varying the potential on the collector C_2 from +40 to −40 volts with respect to T with the help of the potentiometer Z and by observing the secondary current at each stage.

To obtain the neutral atoms of potassium the apparatus was slightly modified. The arc chamber A was replaced by a cylindrical pyrex glass bulb with a side tube containing pure potassium, distilled thrice in stages under vacuum. A Ni disc, 3 cm. in diameter and with a 3 mm. hole at its centre, took the place of the probe and was placed 2 mm. away from and opposite to the filament. Two Ni plates each 1·8 cm. in diameter and with a 3 mm. central hole were fixed behind the disc and replaced the canal, the distance between the disc and the last plate was about 5 cm. The rest of the apparatus was left unaltered. The tungsten filament was maintained at a dull red heat in potassium vapours kept at a temperature of 100–175° c. where the potassium atoms get ionized on striking against the hot tungsten surface and are accelerated to the Ni disc by applying a negative potential to it with respect to the filament. A beam of ions passes out of the hole in the disc and is further defined and collimated by the slit system. As the ions travel through the potassium vapour some of them get neutralized by electron exchange with neutral atoms but retain their initial kinetic energy. The residual positive ions are filtered out of the beam as in the case of Hg particles.

It is assumed that the neutral atoms of K and Hg produced as described above form a homogeneous beam and that all strike the target with the same energy, equal to the accelerating potential of the ions.

§ 3. RESULTS

Many experimental curves have been obtained under different conditions but only typical and essential ones are given here.

Figure 2*a* shows three retarding potential curves for the secondary electrons emitted from Ni by neutral atoms of mercury. Curve (1) was taken for an undegassed target with particles of energy of 2000 v. and with an ion current of 1·1 ma. to the probe while curves (2) and (3) represent results for a well degassed target for particles of energies of 2000 and 1500 v. respectively, the ion current to the probe in both cases being 3·2 ma. The straight lines in figure 2*b* show the relation between the logarithm of the secondary electron current I_s to C_2 and the retarding potential E_R applied on the collector for the corresponding curves in figure 2*a*.

Curves (1) and (2), figure 3 *a*, give results with neutral mercury particles for dirty and clean Mo targets respectively. The energy of the particles in both cases was

2300 v. but the ion current to the probe was 1·4 ma. for the dirty and 3·2 ma. for the well degassed target. The semi-logarithmic curve for the well degassed Mo target is given in figure 3 *b*.

Curves (1) and (2), figure 4 *a*, show results obtained with neutral atoms of potassium of 1500 v. energy for a dirty and a well degassed Mo target respectively. The ion current to the disc was kept the same in both cases. The corresponding $\ln i$–E_R curves are given in figure 4 *b*. Similar curves were obtained for a Ni surface.

It is seen from all the retarding potential curves in figures 2, 3 and 4 that with dirty targets the secondary current to C_2 is saturated at collector potentials 4–16 v. positive with respect to T and at lower voltages with degassed surfaces. In the case of clean Ni targets the collector is positive with respect to T only by a fraction of a volt when saturation occurs. Below saturation potential the current to C_2 decreases with the collector voltage. The secondary electron current reverses and flows from the collector to the target T when the retarding potential on C_2 exceeds a limit. The reverse current increases first as the negative potential on C_2 is raised but soon settles down to a saturated value. Some of the neutral particles incident on the target are reflected back from it and hit the walls of the cylinder C_2, where they liberate secondary electrons which go to the target when the latter is positive with respect to the collector. At a certain negative potential on C_2 this current balances the secondary emission current from T but is greater than this at higher retarding voltages. The reverse current will show saturation when the collector voltage is big enough to stop all the secondary electrons of the target from reaching C_2. The secondary electrons from the target are found to possess all

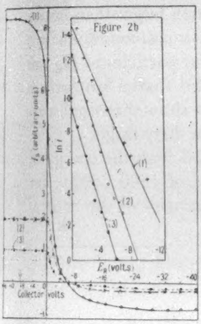

Figure 2 *a*. Results with neutral Hg atoms on Ni.

(1) Target = Dirty Ni.
Probe current = 1·1 ma.
Ion accelerating volts = 2000.

(2) Target = Degassed Ni.
Probe current = 3·2 ma.
Ion accelerating volts = 2000.

(3) Target = Degassed Ni.
Probe current = 3·2 ma.
Ion accelerating volts = 1500.

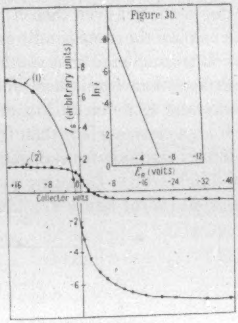

Figure 3 *a*. Results with neutral Hg atoms on Mo.

(1) Target = Dirty Mo.
Probe current = 1·4 ma.
Ion accelerating volts = 2300.

(2) Target = Degassed Mo.
Probe current = 3·2 ma.
Ion accelerating volts = 2300.

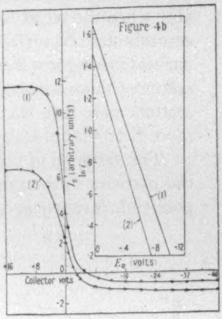

Figure 4 *a*. Results with neutral K atoms on Mo.

(1) Target = Dirty Mo.
Ion current to disc = 1·2 ma.
Ion accelerating volts = 1500.

(2) Target = Degassed Mo.
Ion current to disc = 1·2 ma.
Ion accelerating volts = 1500.

energies in the range 0–20 v. and the energy distribution among them is Maxwellian. Assumption of a similar energy distribution among the secondary electrons liberated from C_2 will explain why the secondary current to the collector does not attain saturation until the positive potential of C_2 with respect to T is greater than a certain value. In the absence of the secondary emission from the walls of C_2 the secondary current to it would show saturation with collector potential equal to or only slightly higher than that of T.

The reverse current is much larger with dirty targets than with clean degassed surfaces. In the case of a dirty Mo target the reverse current is of the same order as the secondary electron current flowing from T to C_2. This decreases appreciably when the target is well degassed, but even then it is much larger than in the case of a clean Ni surface under almost the same conditions, as is seen by comparing curves (2) in figures 2 and 3. It is concluded that the neutral particles of Hg are scattered more from dirty surfaces than from clean ones and that a Mo surface scatters more than Ni. Results with K atoms on Mo and Ni show, however, that more K atoms are scattered from a clean surface than from a dirty target.

The variation of the secondary emission from degassed Ni with the energy of the particles in the range of 1500–2700 v. for mercury and of 1500–2500 v. for potassium is shown in curves (1) and (2) of figure 5. The intensity of the neutral particles in each case was maintained constant throughout the range by drawing a fixed positive ion current to the target at different energies and then filtering out the positives from the mixed beams. Curves (1) and (2) of figure 6 show the change in the secondary emission with the intensity of mercury particles of energy of 2000 v. from clean Ni and Mo targets respectively. Similar results were obtained with K particles on Ni targets.

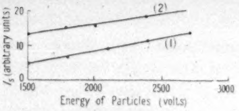

Figure 5.

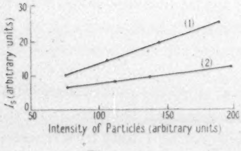

Figure 6.

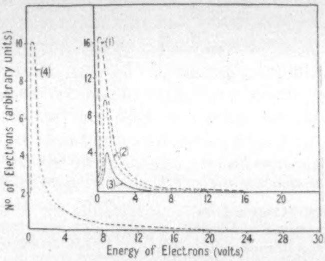

Figure 7.

(1). Undegassed Ni target. Energy of particles = 2000 volts.

(2). Degassed Ni target. Energy of particles = 2000 volts.

(3). Degassed Ni target. Energy of particles = 1500 volts.

(4). Degassed Mo target. Energy of particles = 2300 volts.

The energy distribution among the secondary electrons was obtained for different cases from the corresponding retarding potential curves and is typified by the curves given in figure 7 which are for Mo and Ni targets hit by neutral particles of Hg of varying energies. The distribution in all cases is found to be Maxwellian, the maximum of the curves occurring near zero energy. The position of the peak remains unaltered with variation of the energy of the particles and the condition of the target, but the maximum energy of the secondary electrons increases with the energy of the neutrals.

The secondary emission is always greater from a dirty target than from a clean one and rises almost in proportion to the energy of the particles over the range studied and increases linearly with their intensity as shown in figures 5 and 6.

The secondary electron current in these experiments is of the order of 10^{-8} amp.

The Maxwellian energy distribution among the secondary electrons suggests that the emission is caused by intense local heating of the target spot by the incident neutral particles. The effective temperatures of the target obtained from the semi-logarithmic curves are given below for typical conditions.

Mercury atoms of energy 1500–2500 v.

Undegassed nickel	40 000–45 000° K.
Degassed nickel	29 000–34 000° K.
Undegassed molybdenum	40 000–45 000° K.
Degassed molybdenum	30 000–33 000° K.

Potassium atoms of energy 1200 v.

Undegassed molybdenum	35 000° K.
Degassed molybdenum	29 000° K.

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The Scattering of Fast β-particles through Large Angles by Nitrogen Nuclei

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ABSTRACT. A combined arrangement of counters and a cloud chamber is used to examine the elastic large-scale scattering of β-particles of approximately 1 MeV. energy by nitrogen nuclei. In a total effective track length of nearly 100 km., 131 photographs were obtained in which the scattering angle exceeded 85°. Considering the present results on large-angle scattering together with earlier results on the scattering at smaller angles, the agreement between experiment and Mott's theory is good over the entire angular range 20–180°.

§ 1. INTRODUCTION

It is now generally accepted that the scattering through small angles of β-particles of energy up to a few MeV., by light nuclei, is in agreement with the theory as initially developed by Mott (1929). Most of the earlier experiments were made by the expansion chamber method (Champion 1936); more recent experiments using electrical methods of recording have confirmed and extended these results (Van de Graaff *et al.* 1946). No extensive experiments, however, have hitherto been performed to examine the scattering at angles greater than 90°. At high angles of scattering, the small intensity compared with the general background makes measurement by electron gun methods very difficult. If the expansion chamber method is used in the ordinary way, only a fraction of one per cent of the photographs show large angle deflections. This results in a waste of photographic material and involves tedious and prolonged scrutiny of an enormous number of recordings in the search for rare events. It was therefore decided to fit a standard cloud chamber with counter control in such a way that photographs were obtained only when β-particles suffered deflections greater than about 85°.

§ 2. EXPERIMENTAL ARRANGEMENT

In figure 1 is shown a diagram of the arrangement of the expansion chamber and the counters. Three holes were bored in the top glass plate of the chamber so that the centres of the holes lay on the quadrant of a circle whose centre was at the mid-point of the plate. Into these three holes were inserted three aluminium counters C_1, C_2 and C_3 of length 6 cm. and diameter 2 cm., so that they extended into the full depth of the expansion chamber. The procedure consisted in making expansions at regular intervals of about one minute in the usual way, with the photographic plate permanently exposed. Illumination occurred and a record was obtained only when any one of the counters was discharged by the entry of a β-particle which had been scattered through a large angle.

Accidental discharges of the counters by cosmic rays and other extraneous effects could not be entirely prevented, but illumination during periods other than the sensitive time of the chamber was eliminated by a further control mechanism shown in figure 2. The piston release of the expansion chamber closed K_1 and K_2 at the beginning of an expansion, thus short-circuiting the magnet M_1 and hence releasing the pendulum P_1.

The short time interval between the release of this pendulum and its impact on the key K_3 was the period of sensitivity for β-particle recording. During that time, if the Geiger-counter recorder circuit is closed by the passage of a β-particle through one of the counters in the chamber, the magnet M_2 is short-circuited and the consequent release of the pendulum P_2 operates the flash mechanism for photographing the tracks. Impact of P_1 on K_3 breaks the circuit which, if complete, would allow short-circuit of M_2 should the counter be discharged. Hence at all times other than the short interval between the release of P_1 and its impact on K_3, the flash mechanism is inoperative. In this way, unwanted photographs produced by discharges other than those resulting from a scattered β-particle were reduced to one in forty. Some 1500 expansions were made for every photograph obtained. As shown in figure 1, a lead plate P, which projected 4 cm. into the chamber on one side of the window which admitted the β-particles from a RaE source (placed just outside and fitted with a shutter synchronized with the expansion), prevented the discharge of the counters by β-particles other than those scattered through large angles. A uniform magnetic field was provided by two Helmholtz coils in the usual way and final measurements were made on the images of the tracks by the standard method of reprojection.

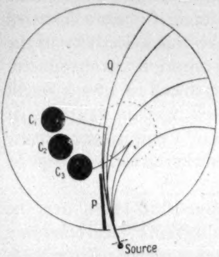

Figure 1.

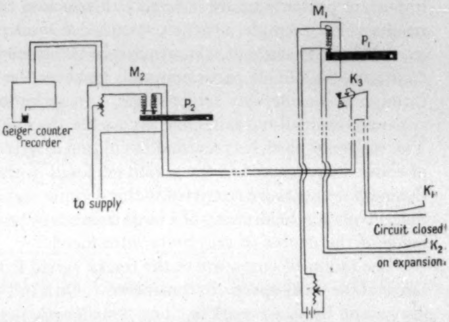

Figure 2.

§ 3. RESULTS

In a total effective track length of nearly 100 km., over 100 photographs were obtained in which β-particles of 0·5 to 1 MeV. energy were scattered by nitrogen nuclei through angles greater than 85°. The improvement in the present arrangement may be judged by considering that in previous work, the tedious examination of 1000 photographs yielded under 1 km. of track and showed only four such large angle collisions. The distribution of the intensity of scattering with angle is shown in the second column of the table. In the third column, the observed values are corrected by a factor which determines the relative chance of recording particles in the given angular range. The last column contains the intensity to be expected theoretically according to Mott's formula for light elements applied to nitrogen.

Angular interval	Observed	Obs. (corrected)	Theoretical
85–100°	74	88	55
100–120°	34	34	37
120–180°	23	23	26
85–180°	131	145	118

§ 4. DISCUSSION

Examination of the table shows that taking the angular distribution as a whole between 85° and 180°, the calculated and the measured intensities are in fair agreement. However, although the present experimental arrangement clearly has considerable advantages over older methods without counter control, certain important precautions are necessary in selecting the data and in interpreting the results. For example, a particle scattered at some point Q in figure 1 would not be recorded if the angle of scattering were 90° since it would not pass through the counters. A similar particle would, however, be registered if it were scattered through a considerably larger angle. In order to make this selectivity of the apparatus as small and as definite as possible, the following criterion was introduced. The scattered particles recorded in column 2 were all produced in a track length of about 4 cm. located in the region enclosed in the dotted circle in figure 1. If the measurements are restricted to this definite region, then by empirical construction, involving the drawing of a large number of typical tracks, an estimate may be made of the degree of selectivity introduced.

The radius of curvature of the tracks varied from about 10 to 14 cm. over the range of the β-ray spectrum considered. On a full scale diagram typical tracks are drawn and these are marked along their length at regular intervals. From these marks branch tracks are drawn representing the bending of the tracks through various angles. Suppose that beyond a certain mark, all tracks bent through less than 90°, for example, no longer meet any one of the counters and are therefore lost. If the total track length from which deflections are measured is y cm. and if the particles bent through the angular range 85–95° are not recorded along a length of more than x cm., the correction factor for these deflections compared with other fully recorded deflections at larger angles is clearly y/x.

Restriction to a definite region also allows the calculation of the effective solid angle subtended by the counters, into which the scattering is recorded. A further point is that since the counters were not touching each other, a certain number of particles passed in the gaps between the counters and were not recorded. These corrections were small for the largest angles but rose to nearly 20% for angles in the region of 90°. It is these factors which account for the difference between the second and third columns of the table.

Counters similar to those used in this experiment had been used in previous work (Widdowson and Champion 1938) and the indications are that for β-particles of the energy and intensity under consideration here, they are 100% efficient. Indeed, the number of particles recorded exceeded somewhat that to be expected theoretically at the smaller angles. It is very unlikely that the discrepancy is real.

It is far more likely that some tracks whose angle of scattering is somewhat less than 85° have been recorded. Since the angle cannot be measured accurately to within a few degrees and the general tendency of the observer is to reject as little data as possible, the fact that the scattering intensity rises very rapidly with decreasing angle could account for the apparent excess at the rather uncertain lower angular limit.

No inelastic collisions of fast β-particles with nitrogen nuclei were recorded with this arrangement, which however, was unsuited to a satisfactory examination of inelastic scattering for two reasons. First, the remaining energy of the electron after collision might be insufficient to penetrate the counter wall, and secondly, the radius of curvature of the track would be reduced after inelastic collision so that it might not traverse the region occupied by the counters. Both these difficulties could be overcome to some extent by using several thin-walled counters distributed over a greater length of arc in the chamber than was used in the present arrangement. However, since such inelastic collisions in nitrogen have been found to be very infrequent it is doubtful if such a straightforward extension of our present experiment would add very much to our knowledge.

Combining these experiments on the large-angle elastic scattering of β-particles of energy up to 1 MEV. by nitrogen nuclei with our previous results on the scattering at smaller angles, we can state that Mott's formula is in good agreement with experiment at all angles of scattering.

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The Penetrating Particles in Cosmic-ray Showers: II. The Lightly-ionizing Penetrating Particles in Penetrating Showers

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MS. received 26 *July* 1948

ABSTRACT. The momentum spectrum of the penetrating particles in a small sample of penetrating showers has been measured. Most of the particles are positive and have momenta of the order 10^9 ev/c. From a study of the interaction of these particles with nucleons in a lead plate it is concluded that whilst a small number may be μ-mesons most are protons, π-mesons, or heavier mesons.

§ 1. INTRODUCTION

An account is given of the lightly-ionizing, penetrating particles occurring in a small group of penetrating showers selected from a larger group of cosmic-ray showers. For the purpose of this analysis a penetrating shower is defined as a shower containing two or more ionizing penetrating particles. Penetrating showers containing single penetrating particles in the cloud chamber have been excluded because of the difficulty of distinguishing them from knock-on showers. A selection has been made, therefore, which is much more rigid than in counter work on penetrating showers.

Broadbent and Jánossy (1947) have suggested that penetrating showers are of two types, namely, local and extensive. A local penetrating shower is one created in the layer of absorber immediately above the counter set while an extensive penetrating shower is one associated with an air shower. It seems probable that the local showers are produced by fast nucleons (Jánossy and Rochester 1943), and it is to this type of shower that the results in this paper mainly refer.

The nature of the penetrating particles in penetrating showers is not yet known but there is much evidence that the local showers contain mesons and protons (Hazen 1944, Daudin 1944, W. M. Powell 1946, Rochester 1946, Rochester, Butler and Runcorn 1947, Rochester and Butler 1947, Fretter 1948). The results presented in this paper, while supporting this conclusion, indicate that only a small fraction of the mesons are μ-mesons.

§ 2. THE EXPERIMENTAL ARRANGEMENT AND THE MEASUREMENT OF MOMENTUM

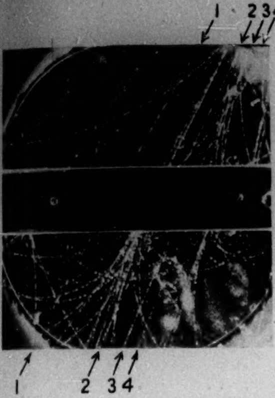
The experimental arrangement was described in an earlier paper (Rochester and Butler 1948, to be referred to as I). All the showers described in the present paper were taken with counter arrangement P, figure 2 of paper I. The thickness of lead in contact with the wall of the chamber and the total thickness of lead above the chamber are given in table 1. There was no lead below the chamber.

As described in I the curvatures of the tracks were measured on the Blackett curvature-compensating machine. Showers were selected with tracks free from

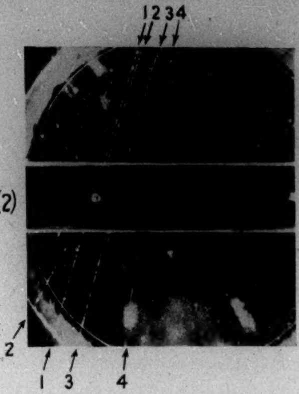
Table 1. Penetrating particles in penetrating showers

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
A(1)	1·8	16·8	7200	1	Positive	1·3	0·1	1·4
				2	Positive	1·1	0·1	1·7
				3	Positive	1·4	0·5	1·3
				4	Positive	0·65	0·8	2·9
B(2)	1·8	16·8	7250	1	Positive	3·3	0·6	0·6
				2	Positive	0·91	12·8	2·1
				3	Positive	1·0	0·8	1·9
				4	Too faint to measure			
C	1·8	16·8	7100	1	Positive	0·66	0·5	2·9
				2	Negative	0·85	2·2	2·2
D	1·8	6·8	7200	1	Positive	1·2	2·4	1·6
				2	Positive	1·1	0·0	1·7
				3	Positive	1·0	0·0	1·9
				4	Positive	0·77	2·4	2·4
E(3)	5·0	10·0	6900	1	Too faint to measure			
				2	Negative	1·1	12·0	1·7
				3	Positive (above)	4·5	28·0	—
					(below)	0·3	—	—
F(4)	5·0	10·0	7300	1	Positive	0·63	3·0	3·0

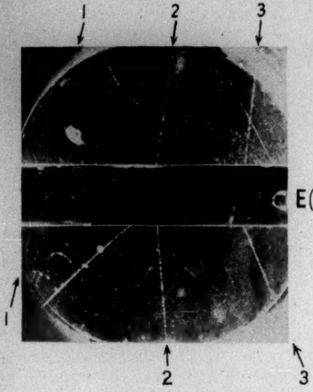
(1) Shower (No. of photograph on Plate). (2) Lead in contact with wall of chamber (cm.). (3) Total thickness of lead above chamber (cm.) (4) Magnetic field (gauss). (5) Number of particle. (6) Sign of particle. (7) Momentum ($\times 10^9$ ev/c.) (8) Angle of scatter (deg.) observed. (9) Angle of scatter (deg.) calculated.



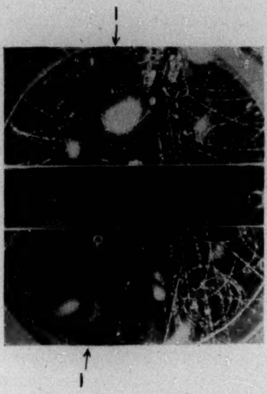
A(1)



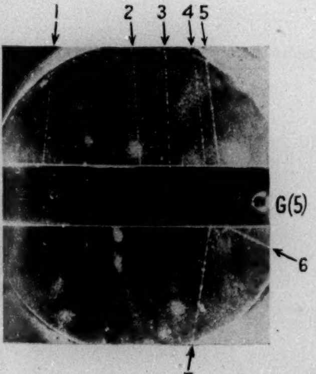
B(2)



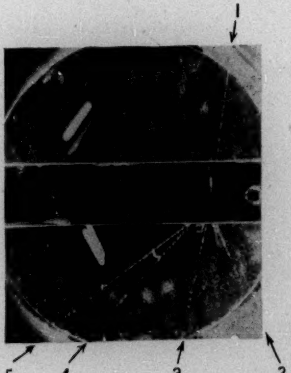
E(3)



F(4)



G(5)



H(6)

PLATE

For all photographs except (3) a positive particle coming down is curved in a clockwise direction.

Photograph (1): A complex penetrating shower consisting of four lightly ionizing penetrating particles and a large electronic shower. The penetrating particles are indicated by numbers. They are positively charged and have momenta ranging from $1\cdot4$ to $0\cdot65\times10^9$ ev/c.
$H=7200$ gauss.

Photograph (2): An example of a simple type of local penetrating shower consisting of four positive penetrating particles whose momenta range from $3\cdot3$ to $0\cdot91\times10^9$ ev/c. Particle 2 is anomalously scattered through $12\cdot8^\circ$ in the lead plate. $H=7250$ gauss.

Photograph (3): A penetrating shower consisting of three penetrating particles 1, 2 and 3. Particle 2 is a negative particle of momentum $1\cdot1\times10^9$ ev/c. and is anomalously scattered through $12\cdot0^\circ$ in the lead plate. Particle 3 is positive with a momentum of $4\cdot5\times10^9$ ev/c. above the plate and $3\cdot0\times10^8$ ev/c. below the plate and is scattered through $28\cdot0^\circ$. The particle below the plate is a proton. The heavily ionizing particle which appears to come from the same region in the lead plate as the lower part of 2 is actually in a plane $1\cdot8$ cm. behind it. Thus if the heavily ionizing particle is connected with particle 2 it must be via an intermediate link. The heavily ionizing particle is positive and has a momentum of $1\cdot6\times10^8$ ev/c. A proton of this momentum ionizes $15\times$minimum whereas the ionization is estimated as $7\times$minimum. The difference may be due to fluctuation or indicate a particle of intermediate mass. $H=6900$ gauss.

Photograph (4): A complex penetrating shower associated with an extensive shower. Only particle 1 is clearly penetrating but at least three other particles in the wide core seem to pass through the plate without multiplication. The shower is coming forward in the chamber at a rather steep angle. Accurate measurement of the tracks is difficult because of the confusion and the rather low technical quality. Most of the particles seem to be positive and some are lightly ionizing with momenta approximately 5×10^8 ev/c. suggesting that they are not protons. The wide-angle pair of tracks at the lower right-hand side of the photograph seem to be those of protons.
$H=7300$ gauss.

Photograph (5): An unusual shower consisting of several high-energy particles which stop in the lead plate without producing visible particles. All the particles except 3 are well in the illuminated region of the chamber and nearly all have momenta much above 10^9 ev/c.
$H=7100$ gauss.

Photograph (6): An example of a high-energy star induced by particle 1. Particles 4 and 5 have momenta of approximately 10^9 ev/c. and are positive and negative respectively. This photograph would appear to indicate the creation of mesons or protons. $H=6600$ gauss.

obvious distortion and no track was measured unless its length in the chamber was greater than 6 cm. A check on the performance of the cloud chamber was made in two ways, firstly, by measuring up 87 meson tracks photographed without magnetic field, and secondly, by measuring up 90 meson tracks in the magnetic field. All these tracks were taken under chamber conditions identical with the showers. The lead above the chamber was, however, removed. It was found that a fairly good Gaussian curve could be fitted by the method of least squares to the no-field curvature measurements. The half-width of the Gaussian corresponded to a momentum of $8{\cdot}4 \times 10^9$ ev/c. in a magnetic field of 7500 gauss, which may be taken as the maximum detectable momentum (Blackett 1937).

The meson tracks in the magnetic field provided an effective check of the performance of the cloud chamber under conditions identical with those for penetrating showers. The resulting meson spectrum agreed closely with the data given by Wilson (1946). It may therefore be assumed that the performance of the cloud chamber was satisfactory.

§ 3. THE PENETRATING PARTICLES IN PENETRATING SHOWERS

(i) *Photographs and classification of the showers*

Using the criteria outlined in §§ 1 and 2, six penetrating showers containing sixteen penetrating particles have been selected for measurement and the results are given in table 1. The number in brackets after the letters in column (1) of table 1 refer to the photographs on the plate. A photograph of shower D has already been published (shower 2 of Rochester and Butler 1947). With the photograph published by Rochester, Butler and Runcorn (1947) these photographs present the main types of penetrating shower in a magnetic field. It is seen that a broad classification is possible in that some penetrating showers consist only of a few ionizing penetrating particles as, for example, showers B, D and E, while others include a considerable electronic component, as for example, showers A and F. This classification is clearly shown in the investigation made by Rochester (1946) and by Bridge, Hazen and Rossi (1948). Showers of the first type are presumably local penetrating showers, for in each case the penetrating particles can be traced back to a common point of origin in the absorber. Showers of the second type may be associated with extensive showers. Shower F was connected with an extensive air shower but it is not possible to classify shower A because the counter extensive tray was not in operation when this photograph was taken.

(ii) *Positive excess and spectrum*

The data show several interesting features, two of the most striking being the large positive excess and the relative abundance of particles with momenta below 2×10^9 ev/c. It will be noticed that of the 16 penetrating particles, 14 are positive and only 2 negative. This large positive excess suggests that some of the penetrating particles are protons. However, with such a small sample

Table 2. Spectrum of penetrating particles

Momentum range ($\times 10^8$ ev/c.)	1–5	6–10	11–20	21–∞
No. of penetrating particles.	0	8 ± 3	6 ± 2	2 ± 1

the result should be taken with caution. Nevertheless it is interesting to see how many of the particles are probably *not* protons. We may assume that the negative particles are not protons and if we add the particles which are still lightly ionizing when their momenta are less than $0{\cdot}66 \times 10^9$ ev/c.—a proton of this momentum would ionize twice minimum—we find five out of the sixteen particles.

(iii) *Anomalous scattering*

Another striking feature is the anomalous scattering of three of the penetrating particles in the lead plate. These particles are: particle 2 of shower B, which is scattered through 12·8°, and particles 2 and 3 of shower E, which are scattered through 12·0° and 28·0° respectively. The probable angle of scatter for any one of these particles, calculated from Williams' formula, is not greater than 2°. There is little doubt that this result is significant especially when taken in conjunction with previous work on penetrating showers. Thus, Rochester (1946) found two cases of particles scattered through 10·2° and 18·0° respectively out of 32 penetrating particles in penetrating showers. Although the momenta of the particles were not known, the type of shower was so similar that it may be assumed that these were also cases of anomalous scattering. In all, five cases of scattering have been found in 48 penetrating particles or, in terms of the thickness of lead traversed, one particle per 25 cm. lead. This value is in sharp contrast with the scattering of ordinary mesons. Thus Wilson (1940) found one case of anomalous scattering per 40 m. of lead traversed, and Shutt (1946) and Code (1941) have obtained similar results. Where the sign could be determined almost all of the anomalously scattered particles were positive. It has therefore been suggested that the observed cases of anomalous scattering are due to protons and that the observed scattering cross-section does not represent the true cross-section for mesons. The results presented here support this conclusion; indeed, particle 3 of shower E can be unambiguously identified below the plate as a proton. It is not, however, valid to assume that the particles above and below the plate are necessarily the same, nor even that they are all protons. They could, for example, be protons or π-mesons which produce stars in the plate with secondary particles of such short ranges that all are absorbed except one. It is improbable that the scatterings represent examples of the spontaneous disintegration observed by Rochester and Butler (1947) in shower D, because only one such event has been found in the gas of the chamber, whereas at least five times as many events should have been observed in the gas as in the lead plate. Thus, it must be concluded that the scatterings are due to a collision process and not to any type of spontaneous process.

The minimum momenta of a meson of mass 200 m_e and a proton to penetrate the lead plate are $1{\cdot}4 \times 10^8$ ev/c. and $5{\cdot}7 \times 10^8$ ev/c. respectively. A proton of this momentum would ionize 2·4 times minimum.

§ 4. SHOWERS G AND H

In this paragraph a description will be given of two unusual showers which are shown in photographs 5 and 6 of the plate. The data for these showers are given in table 3. The positions of the ends of the tracks with respect to the front of the chamber, which is 9 cm. in depth, are given in columns (8) and (9), so that some idea of the space-orientation of the tracks can be obtained. The lengths of the tracks vary from 6 cm. to 12 cm. in the chamber.

(i) *Shower G*

This shower was connected with an extensive air shower and like showers F and E consists mainly of a collimated group of positive particles. The tracks are, however, not quite parallel, but project back to a common point of origin in the lead absorber above the counter set (see figure 2, paper I). It is clear that these particles are not electrons for most have momenta greater than 10^9 ev/c. and yet do not produce showers in the lead absorbers either above or in the cloud chamber. According to Chakrabarty (1942) an electron of momentum 10^9 ev/c. produces on the average a shower of 10 particles in a lead plate of 3 cm. thickness and an electron of 5×10^9 ev/c. a shower of 50 particles. The particles seem to be similar to the penetrating particles discussed in § 3. Particle 5 is scattered in the plate and emerges as particle 6; particle 7 is probably associated with the same collision process. The other particles are remarkable in that they are apparently completely absorbed in the lead plate without producing other charged particles. Although some are moving slightly backwards, their inclinations and positions at the plate do not suggest that they would have gone out of the illuminated region. Just as in the case of the large-angle scattering it is not known what type of collision process will account for this phenomenon. A large star, could however, be produced in the plate and no visible particle appear.

Table 3. Showers G and H

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
G(5)	5·0	10·0	7100	1	Positive	0·75	4·9	3·8
				2	Positive	7·0	4·0	2·9
				3	Positive	8·5	6·0	4·0
				4	Positive	2·3	2·8	2·8
				5	Positive	1·7	4·0	4·0
				6	—	—	3·5	3·5
				7	—	—	4·5	4·5
H(6)	5·0	10·0	6600	1	Positive	5·2	3·3	3·3
				2	—	—	2·8	2·8
				3	—	—	2·8	2·8
				4	Positive	1·5	2·1	1·3
				5	Negative	1·1	4·0	6·2

(1) Shower (No. of photograph on Plate.) (2) Lead in contact with wall of chamber (cm.). (3) Total thickness of lead above chamber (cm.). (4) Magnetic field (gauss). (5) Number of particle. (6) Sign of particle. (7) Momentum ($\times 10^9$ ev/c.). (8) Position of track at plate (cm.) (9) Position of other end of track (cm.).

This shower and several of the others, for example, showers B, E and F also show another very interesting phenomenon, namely, tracks which are almost parallel in the cloud chamber. In at least three of the cases the tracks project rather accurately back to the lead absorber which is 75 cm. above the centre of the cloud chamber. It is thus quite possible that the showers are originally star-like and that geometrical factors cause the selection of an almost parallel core of particles in the cloud chamber. Low-energy penetrating particles, if not lost by their emission at wide angles, may be lost by absorption in the lead immediately above the cloud chamber. Thus the showers presented here are quite consistent with the star-like penetrating showers found by Hazen, Fretter and others. Moreover, they are also consistent with production by incident particles whose ranges are from 5 to 10 cm. of lead. In order to account for showers like E, F and G, however, where particles emerge from a block of lead 5 cm. in thickness and are then absorbed in a further 3 cm. lead plate, we require that many of the secondary particles are, like the primary particles, catastrophically absorbed with a range of from 5 to 10 cm. lead.

(ii) *Shower H*

This shower is of especial interest because it is an explosive-type shower in which high-energy particles are emitted at large angles to the direction of the incident particle. It will be seen from table 3 that the incident particle 1 has a very high energy. The particles which emerge below the plate come from a point 1 cm. from the bottom of the plate. The quality of tracks 2 and 3 is not high and they are so short that no measurement of their energies has been attempted. Tracks 4 and 5 are of better quality and are sufficiently long to permit of fairly accurate momentum measurements. The results of these measurements are surprising. They show that these particles are of opposite sign and have almost the same momenta (i.e. 10^9 ev/c.). These particles make angles of 35° and 46° respectively with the direction of the incident particle. Clearly, they are not electrons since two electrons of such high momenta are unlikely to emerge singly after traversing such a large thickness of lead. Again, the particles are unlikely to be heavier than protons. Thus, for each of the particles the ionization is typical of a fast particle and therefore the rest energies must be equal to or less than the momenta. Since the momenta are approximately 10^9 ev/c. the rest masses must be approximately equal to or less than the mass of the proton. If the particles are protons one is a negative proton. They may, however, be particles of intermediate mass. The large angle of emission does not help in deciding what kind of heavy particle, however, for with a suitable model a wide range of particles can be emitted at large angles to the direction of the incident particle. This shower raises many interesting questions, for example, what is the mechanism of the transfer of such a large amount of momentum, why are there so few low-energy particles emitted, and why are there so few mesons emitted? The answers to these questions await a fuller understanding of the interaction of fast nucleons.

§ 5. CONCLUSION

The following are the main results which follow from the analysis of the small sample of penetrating showers described in this paper. (i) A large fraction of the ionizing particles consists of particles which are different from ordinary mesons. Thus, out of twenty particles, with momenta greater than 10^9 ev/c., four are anomalously scattered in the 3 cm. lead plate and three produce no visible ionizing particle. (ii) A number of the positive particles are likely to be protons. This conclusion is suggested by the positive bias of the spectrum and by the presence of identifiable protons in the showers (for example, in shower 2 of Rochester and Butler 1947). (iii) The remaining penetrating particles may be either π-mesons (Lattes, Occhialini and Powell 1947) or heavier mesons (Rochester and Butler 1947). (iv) Some of the showers are very complex. There are complex showers which consist mainly of electrons and penetrating particles, and others which consist of large collimated groups of positively charged penetrating particles.

ACKNOWLEDGMENTS

It gives us great pleasure to express our sincere thanks to Professor P. M. S. Blackett, F.R.S. for the excellent facilities which he has given us and for the keen interest he has taken in this investigation. We are indebted also to Dr. S. M. Mitra and Mr. W. G. Rosser who operated the cloud chamber during part of the experiment and took several of the photographs described in this paper.

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A few Remarks about Spectroscopy at Low Frequencies

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Being the man who has just failed to make the important discoveries in radio- and microwave spectroscopy and having little to report about recent researches (Dr. Bloembergen will speak for himself) I shall say a few words about the reasons I had about twelve years ago to expect that it was possible to start spectroscopy at low frequencies.

It was clear that electric dipole lines would be broadened enormously by interaction and, moreover, often would not be allowed by the selection rules. So I came to consider the magnetic dipole transitions between the Zeeman levels of paramagnetic ions and between the nuclear Zeeman levels, and arrived at the following result.

If the absorption in the centre of the absorption band is described by the imaginary part χ'' of the magnetic volume susceptibility, we have $\chi'' \simeq \chi_0 \nu/\Delta\nu$, where $\Delta\nu$ denotes the width of the band and χ_0 the susceptibility connected with it. Often χ_0 is the static susceptibility, $\chi_0 = NM^2/3\boldsymbol{k}T$, where N is the number of magnetic moments M per unit of volume. In relaxation bands $\Delta\nu \simeq \nu$, but in narrow resonance lines we may win a factor 10^5 or more. For paramagnetic substances at room temperature we have $\chi_0 \simeq 10^{-5}$ and if $\Delta\nu \simeq \nu$ we get $\chi'' \simeq 10^{-5}$. For nuclei χ_0 is at least 10^5 smaller but we may win a large factor $\nu/\Delta\nu$ back for resonance lines.

The optical absorption coefficient in $I = I_0 \exp(-\alpha x)$, where I is the intensity in a plane wave, is

$$\alpha = 8\pi^2\nu\chi''/\mathbf{c}, \qquad \ldots\ldots(1)$$

which for $\chi'' = 10^{-5}$ and $\nu = 10^7$ becomes $\alpha \simeq 3 \times 10^{-7}\,\mathrm{cm}^{-1}$. Thus an absorption path of many kilometres would be needed for observation. In the normal optical region we are a factor 10^8 better off because of the larger ν while the extra advantage of the larger electric dipole probabilities is often outweighed by smaller densities.

At normal radio frequencies it is not difficult to measure $\chi'' = 10^{-5}$ either by bridge methods or by superheterodyne methods, while caloric measurements with an amplitude H_0 of 10 oersted permit observation of $\chi'' = 10^{-7}$ to 10^{-9} dependent on the temperature.

After the war it was found that with microwaves one could also easily reach $\chi'' = 10^{-5}$ while Purcell's refined bridges allow us to measure $\chi'' = 10^{-7}$ to 10^{-8}. The best accuracy at room temperature, however, was obtained by Zavoisky who apparently reached $\chi'' = 10^{-9}$ but who did not publish his method of observation in detail.

Finally I may quote from my old notes that in the case of a single spin the number n of transitions per second is

$$n = 4\pi M^2 H_0^2 / 3\mathbf{h}^2 \Delta\nu. \qquad \ldots\ldots(2)$$

Taking $\Delta\nu = 10^4$ and $H_0 = 10$ oersted we obtain $n = 2 \times 10^5\,\mathrm{sec}^{-1}$ for hydrogen nuclei. This led to the suggestion of experiments with molecular beams.

Paramagnetic Resonance in Salts of the Iron Group—A Preliminary Survey: I. Theoretical Discussion*

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ABSTRACT. By means of paramagnetic resonance the energy levels of paramagnetic salts can be investigated in much greater detail than by any previous method. Using centimetre wavelength radiation it is possible to determine small splittings of the levels and their effective "*g*-value", and to isolate absorption lines due to different ions in unit cell. The interpretation of such effects in salts of the iron group by crystalline electric field theory is outlined.

The contributions from spin–spin and spin–lattice interaction to the widths of the lines are discussed. In the ions $3d_1Ti^{+++}$, $3d_2V^{+++}$, $3d_6Fe^{++}$ and $3d_7Co^{++}$, the spin–lattice relaxation time may be extremely short, and the lines correspondingly broad at room temperature; by working at sufficiently low temperatures, however, it should be possible to make the effects of spin–lattice interaction negligible in comparison with those of spin–spin interaction.

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* This work was discussed at the Physical Society Conference at Oxford on 23rd–24th July 1948

§1. INTRODUCTION

THE use of a radio-frequency magnetic field to induce transitions between the different energy levels corresponding to the orientations of a nuclear spin in a magnetic field was first suggested by Gorter (1936), who attempted without success to find the corresponding absorption line. Using the molecular beam technique, Rabi and his co-workers (1939) applied the method to the determination of nuclear gyromagnetic ratios with outstanding results. Direct observation of resonance absorption and dispersion in liquids and solids was accomplished independently by Purcell, Torrey and Pound (1946) and Bloch, Hansen and Packard (1946). High precision measurements of nuclear *g*-values and much information on line breadths and thermal relaxation times have already resulted from this work.

Application of the same principles to electron spins instead of nuclear spins produces the analogous phenomenon of paramagnetic resonance. The energy absorbed by a paramagnetic salt from an oscillating magnetic field of a given frequency is measured as a function of a variable steady field applied at right angles to the alternating field. An absorption line is observed whenever the separation of the energy levels on the paramagnetic ion is equal to the quantum of energy corresponding to the radio frequency. By making measurements over a range of frequencies the transitions can be plotted as a function of the applied field. In this way the behaviour of the energy levels can be studied directly, and much more detailed information can be obtained than by any previous method, such as susceptibility and specific heat measurements, paramagnetic rotation and relaxation.

The purpose of this and the following paper is to present a preliminary survey of the method of paramagnetic resonance, with particular reference to salts of the first transition group. Measurements have been made on over a hundred salts, and in the first approximation, the results show that the determining factor is the number of electrons in the ion. These results, together with some discussion of each ion, are presented in the second paper. The first paper gives an outline of the effects of the crystalline electric field on the ions, together with a discussion of the question of line width, especially in connection with the spin–lattice relaxation time.

§2. THE ABSORPTION SPECTRUM

The fact that most paramagnetic salts of the iron group have susceptibilities conforming closely to those to be expected from electron spin only has been explained by the electric crystalline field theory developed by Bethe (1929), Van Vleck (1935) and others. Not only do the orbital and spin momentum vectors **L** and **S** fail to form a resultant **J**, but the degeneracy of the $2L+1$ orientations of the orbital momentum is partly or wholly lifted according to the symmetry of the electric field. The separations between these orbital levels may amount to some 10^4 cm^{-1} and the orbital momentum is therefore "quenched" at ordinary temperatures. The electron spin has no direct interaction with the electric field of the lattice, however, and therefore remains substantially "free", and the salts obey Curie's law (or a Weiss law) quite well at room temperature. In a magnetic field the degeneracy corresponding to the $2S+1$ orientations of the spin is lifted, and resonant absorption lines can be observed corresponding to transitions between these different levels. They may be detected by the damping of a tuned circuit containing the paramagnetic salt, as in the analogous nuclear resonance phenomenon. Owing to the large magnetic moment associated with the electron the energy levels will be correspondingly broader than in the nuclear case and are not sufficiently separated until fields of the order of kilogauss are applied. The absorption lines will lie then at centimetre wavelengths.

If the spin of the paramagnetic ion is perfectly free its $2S+1$ levels will have, in a magnetic field H, energies $-(g\beta H)M_S$, where M_S has the values $S, S-1, \ldots -S$. Here g is the Lande "g-factor" and β is the Bohr magneton. When an alternating magnetic field is applied at right angles to H, transitions in M_S of the type $\Delta M_S = \pm 1$ are allowed. Equating the energy separation of adjacent levels to a quantum of energy ($\boldsymbol{hc}\bar{\nu}$) one finds that an absorption line should occur at a wavelength of

$$\lambda = \frac{1}{\nu} = \frac{\boldsymbol{hc}}{g\beta H} = \frac{4\pi \boldsymbol{mc}^2}{\boldsymbol{e}(gH)}. \qquad \ldots\ldots(1)$$

If $g=2$, and H is measured in kilogauss, this may be written $H\lambda = 10{\cdot}7$. Zavoisky (1945, 1946) and Cummerow, Halliday and Moore (1947) have shown that several salts of the iron group exhibit absorption maxima at centimetre wavelengths in magnetic fields close to that predicted by this equation.

In general the paramagnetic ion is not perfectly free, owing to a residual spin–orbit coupling, which produces the following effects: (*a*) the effective g-value is no longer exactly two; (*b*) when $S > \frac{1}{2}$, the levels may be split in zero magnetic field, with separations of the order of $1\,\mathrm{cm}^{-1}$; (*c*) the crystal is magnetically anisotropic, and the behaviour of the spin levels in a magnetic field depends on the orientation of the latter relative to the axes of the crystalline field; (*d*) when the unit cell of the crystal contains two or more paramagnetic ions, each subject to a differently oriented though otherwise identical crystalline field, it follows from (*c*) that the paramagnetic resonance spectrum of each ion will be different for an arbitrary orientation of the crystal.

When all these effects are present, the absorption spectrum of the salt will be very complicated and the interpretation difficult. In some cases sufficient data are available from x-ray measurements to deduce the nature of the crystalline field, and hence to calculate the behaviour of the energy levels in a magnetic field. A detailed comparison of experiment with theory is then possible, as, for example, in the case of chrome alum (Bagguley and Griffiths 1947, Weiss, Whitmer, Torrey and Hsiang 1947). These measurements show how strongly dependent the paramagnetic resonance spectrum is on the orientation of the crystal, even when the anisotropy of the susceptibility is very small. In cases where the exact nature of the crystalline field is not known, it is still possible to determine an initial splitting by plotting a transition back to zero magnetic field, as in ammonium chrome alum at low temperatures (Bleaney and Penrose 1948). It is not the purpose of this paper to consider particular salts in detail, however, but to assess the general advantages and limitations of the method of paramagnetic resonance in connection with the results reported in the following paper.

§ 3. THE BREADTH OF PARAMAGNETIC RESONANCE LINES

One of the chief limitations of the method of paramagnetic resonance lies in the lack of resolution of the fine structure of the spectrum owing to the breadth of the lines. The line broadening is associated with the interactions of the electron spins with each other and with the lattice; these interactions have been studied at lower frequencies by means of the phenomenon of paramagnetic relaxation by Gorte and others (for a general review see Gorter 1948). The two interactions, spin–spin and spin–lattice, are somewhat different in nature and it is convenient to consider them separately.

Spin–spin interaction

In addition to the indirect effects of the crystalline field, each spin is subject to an interaction with the neighbouring spins, as a result of which the energy levels are spread out into a band. The continual exchange of energy between the various spins gives a lifetime for each spin in a given state of about 10^{-9} seconds, and at lower frequencies the phenomenon has been studied from the point of view of the non-resonant absorption connected with this relaxation time. Since paramagnetic resonance lines are generally observed at constant frequency in a variable magnetic field, it is more convenient to express their widths in terms of field, and the effect of spin–spin interaction may be regarded in the following way. Each ion is subjected not only to the external magnetic field, but also to the field of its neighbouring dipoles. In a weak field these are (classically speaking) randomly oriented, and there results a spread in the field acting on the individual dipoles, of the order μ/r^3, where r is the mean distance between neighbouring dipoles. This "internal field" will give rise to a line width strongly dependent on the degree of "magnetic dilution"; even in such dilute salts as the alums the line width will amount to a few hundred gauss. One may expect a considerable change in line breadth with the orientation of the crystal in the applied magnetic field, as (to a first approximation) only the component of the local field in the direction of the applied field matters; the contribution from each neighbouring dipole to this will have an angular dependence of the form $(1-3\cos^2\theta)$ where θ is the angle between the line joining the dipoles and the applied field. In general, however, the width will be independent of the applied field (so long as saturation effects are negligible) and of the temperature.

The theory of the dipole–dipole interaction has been formulated by Van Vleck (1937) who has applied it to the calculation of line width in a forthcoming paper (Van Vleck 1948). An approximate method for the analogous nuclear case is outlined by Bloembergen, Purcell and Pound (1948). The calculation for paramagnetic salts is considerably more complicated, however, owing to the anisotropy and splitting of the energy levels caused by the crystalline electric field.

In addition to pure dipole–dipole coupling, it has generally been assumed that some kind of exchange interaction occurs between the ions. The theory of exchange forces in paramagnetic salts is in its very early stages, but it appears necessary to distinguish between an isotropic exchange and an anisotropic exchange force. The former is equivalent to a simple coupling between the spins proportional to the cosine of the angle between them, as in the Heisenberg theory of ferromagnetism. Gorter and Van Vleck (1947) point out that such exchange forces should cause a narrowing in the centre of the line and a broadening in the wings. Pryce (1948) has shown that in copper sulphate these forces, acting between the two different ions in unit cell can prevent the resolution of separate absorption lines due to the two ions, unless high magnetic fields are used. The mechanism of the anisotropic exchange is uncertain, though it has been applied by Opechowski (1948) to the explanation of the abnormally large specific heat of copper potassium sulphate below 1° K. discovered by de Klerk (1946).

Spin–lattice interaction

From the theory of paramagnetic relaxation it follows that when the magnetic field acting on a paramagnetic salt is changed, the magnetization M does not instantaneously take up the new value appropriate to the field, but approaches it exponentially with a time constant $\rho_L/2\pi$. The change of magnetization requires that the energy of the spin system change, which can only happen by means of an exchange of energy with the lattice. This exchange is, of course, continuous, so that the life of each paramagnetic ion in a given energy state is finite. This is true even when the external magnetic field is constant, since then the spins are only in a state of dynamic equilibrium which makes the total magnetic moment of the whole spin system constant when averaged over a time long compared to the relaxation time. When a radio-frequency field, whose period is comparable with or shorter than the relaxation time, is used in conjunction with a large constant field, the finite life time becomes manifest, though in different ways according to whether the alternating field is parallel or perpendicular to the constant field.

In the case of parallel fields, the addition of a small alternating field causes a first order fluctuation in the magnitude of the constant field, and hence, at low frequencies, a corresponding first order fluctuation of the magnetization. As the frequency increases, the spins are no longer able to exchange energy with the lattice sufficiently rapidly and the alternating component of the magnetization decreases. This decrease in the susceptibility is accompanied by a non-resonant absorption which is a maximum at a frequency equal to $(1/\rho_L)$. In the case of perpendicular fields, the vector resultant of a small alternating field and the large constant field contains only a second order fluctuation in magnitude. Consequently no exchange of energy between spin and lattice is required and there is no corresponding dispersion of the non-resonant type. The finite life-time of an ion in a given energy state due to spin–lattice interaction will, however, contribute to the width of the paramagnetic absorption line in addition to the width due to spin–spin interaction. When the spin–lattice relaxation time is long compared with the spin–spin relaxation time the additional width will be negligible, but when the former becomes comparable with the latter there will be a noticeable increase in line breadth. This point will be discussed in more detail later.

The effects of spin–lattice and spin–spin interaction may be summed up as follows:—

Case 1. No constant magnetic field: non-resonant absorption at frequencies of the order $1/\rho_S$ (c/s.).

Case 2. Constant magnetic field parallel to alternating field: non-resonant absorption at frequencies of the order $1/\rho_L$ and $1/\rho_S$ (c/s.).

Case 3. Constant magnetic field perpendicular to alternating field: resonant absorption at frequencies corresponding to separation of the energy levels. Line breadth of the order $(1/\rho_S+1/\rho_L)$ (c/s.).

§4. DISCUSSION

In certain cases it may happen that no paramagnetic resonance lines are observed, even at wavelengths and magnetic fields corresponding to the relation given in equation (1). There are two important reasons which must be considered in connection with this failure: (i) the spin levels may be split so that no two levels between which transitions are allowed have the right separation ($hc\bar{\nu}$) for the frequency of the measurement at a practicable value of the magnetic field; (ii) the width of the absorption lines may be so great that no detectable change in the absorption occurs in the fields used. It is convenient to consider these two cases separately.

(i) By an important theorem of Kramers (1930), whenever an ion contains an odd number of electrons, the degeneracy of the spin levels can never be lifted entirely by an electric field. At least a two-fold degeneracy must remain, which can be resolved by a magnetic field. Paramagnetic resonance would therefore be expected in the ions with an even spin multiplicity, but in the remainder the separation in zero magnetic field may be too great to allow them to converge sufficiently in the fields at our disposal. The phenomenon should, however, appear at frequencies comparable with this initial splitting.

In this connection it is important to note that this splitting depends on λ^2/Δ where λ is the coefficient of the spin–orbit coupling $\lambda(\mathbf{L}.\mathbf{S})$ and Δ is the splitting of the lowest orbital levels by the crystalline field (Van Vleck 1932).

(ii) The estimated line width H_i due to spin–spin interaction amounts to some hundreds of gauss in hydrated paramagnetic salts. In the unhydrated salts the magnetic dipole field will be somewhat larger, but the effects of exchange forces are appreciable (e.g. metamagnetic curie points of anhydrous chlorides) and the paramagnetic resonance lines may be narrowed in consequence.[*] Thus the width due to spin–spin interaction should never be great enough to account for failure to detect resonance if magnetic fields up to 10 000 gauss are used.

The line width due to spin–lattice interaction is difficult to calculate. There is no direct coupling between the spins and the lattice, and two indirect couplings must be discussed. The first of these, originally pointed out by Waller (1932), acts through the magnetic dipole interaction of the spins, which varies periodically owing to the spatial vibrations of the ions under the action of the lattice waves. Through this coupling the spins can make transitions either by exchanging a quantum with a lattice vibration of the appropriate frequency, or by "scattering" a lattice quantum with change of energy ("Raman process"). Though the latter is a second order process, all lattice quanta can take part, whereas in the former only a limited number are available. In general the former process is the more important at helium temperatures, but at liquid oxygen temperatures the "Raman" processes should predominate.

The coupling mechanism considered by Waller leads to relaxation times longer than those observed experimentally, which in most salts are of the order of 10^{-6} second at 90° K. Further, in a number of cases, the failure to detect relaxation makes it necessary to suppose that the relaxation time is shorter than 10^{-8} second. An alternative coupling mechanism was proposed by Kronig (1939). Through the action of the lattice waves, fluctuating components of the crystalline electric field are set up which act on the orbital motion of the electrons, and hence cause transitions in the spin levels via the spin–orbit coupling. This mechanism is particularly potent when the orbital motion is only partly "quenched", that is, the splitting of the lowest orbital levels by the crystalline field is small. Using a simplified model, Kronig obtained the following estimates for the relaxation time in the case of $S=\frac{1}{2}$, the interchange of energy with the lattice vibrations being again either simple exchange of quanta or by the "Raman" process.

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[*] An interesting example is anhydrous chromic chloride, $CrCl_3$, on which measurements have been made at 3 cm. wavelength. The value of H_i is about 1500 gauss, but even in the powder the half-width at half-intensity of the resonance line is about 50 gauss at room temperature! Since the crystals are probably anisotropic, somewhat narrower lines may be expected in a single crystal. At 20° K. the width is somewhat greater, but on cooling below the Curie point (16° K.) the line becomes very broad (about 2000 gauss) and the absorption is appreciable even in zero magnetic field.

Simple exchange: $\rho_{\mathrm{L}} \simeq \dfrac{10^4\Delta^4}{\lambda^2 H^4 T}$ (sec.) "Raman" scattering: $\rho_{\mathrm{L}} \simeq \dfrac{10^4\Delta^6}{\lambda^2 H^2 T^7}$ (sec.)

In these formulae the temperature T is assumed small compared with the characteristic Debye temperature of the lattice, and Δ, λ are in cm^{-1}.

Since the effect of spin–lattice relaxation in broadening paramagnetic resonance lines is superimposed on that of spin–spin relaxation, for which the characteristic time is about 10^{-9} second, it follows that the effect will be negligible unless ρ_{L} is of the order of 10^{-9} second or shorter. To broaden the lines so much as to make them unobservable would require ρ_{L} to be as small as 10^{-11} second. No experimental data are available from which relaxation times of this order can be deduced, while the theory of Kronig is confined to the case of $S=\frac{1}{2}$. Van Vleck (1940) has made detailed calculations for chrome alum and caesium titanium alum, and for the "Raman" processes in the latter he obtains ($T \ll$ Debye lattice characteristic temperature) $\rho_{\mathrm{L}} \simeq 10^{-15}\Delta^6/T^9$ (sec.), independent of H so long as it is greater than the internal field. Thus if Δ were as small as $100\ \mathrm{cm}^{-1}$, ρ_{L} would be 10^{-12} second even at 10° K. This case is probably exceptional, but from these calculations one may infer that the relaxation time will vary with a high power of the crystalline splitting of the lowest orbital levels, and with a high inverse power of the temperature. From this last it follows that by lowering the temperature it should be possible to make the spin–lattice relaxation time long enough to permit the observation of paramagnetic lines, and ultimately to reduce their width to that due to spin–spin relaxation alone.

To sum up, one may say that from both (i) and (ii) it appears that the salts in which paramagnetic resonance may not be observed are those in which the spin–orbit coupling is large, and more particularly those in which the splitting of the lowest orbital levels by the crystalline field is small. In general this means those paramagnetic ions in which an electric field of cubic symmetry leaves a degenerate orbital level at the bottom, the degeneracy being lifted only by a field of less symmetry. It should be possible to observe paramagnetic resonance even in these salts, however, by working at sufficiently high frequencies, when there are initial splittings, and at sufficiently low temperatures, when the spin–lattice relaxation time is abnormally short.

§ 5. SALTS OF THE IRON GROUP

The potential of the crystalline field is generally assumed to be of the form $V = Ax^2 + By^2 - (A+B)z^2 + D(x^4+y^4+z^4)$ though in some cases the axes of the second order (rhombic) field may be rotated with respect to those of the fourth order (cubic) field. Bethe (1929) has used group theory to show how the orbital levels split up in these fields, and an important point concerns which way up the multiplet appears, this depending for each ion on the sign of the coefficient D. Gorter (1932) has pointed out that if the ion is surrounded by six oxygen ions (negatively charged) arranged in an octahedron, the sign of D will be positive, whereas it will be negative if the arrangement is four in a tetrahedron or eight in a cube. In the following discussion the octahedral arrangement will be assumed, and the splitting of the orbital levels given accordingly.

The free ions of the first transition group are all in D-, F-, or S-states. In the latter case ($3d^5Fe^{+++}$, Mn^{++}) there is no orbital momentum to be quenched, and the crystalline field has only a very small effect on the spin levels. In a cubic field a D-state is decomposed into a doublet and a triplet, while an F-state resolves into a singlet and two triplets. The doublet state is non-magnetic, and when this is lowest, the splittings which are involved in the perturbation of the spin levels are those between the lowest level of the doublet and the triplet levels. Thus in this case, and in an F-state when the singlet lies lowest, the orbital levels are already sufficiently split up in a cubic field, the separation being of the order of 10^4 cm^{-1}, for the perturbation of the spin levels to be small. This applies to the ions $3d^3Cr^{+++}$, $3d^4Cr^{++}$, $3d^8Ni^{++}$ and $3d^9Cu^{++}$, in all of which paramagnetic resonance is observed at room temperature (see § 3 of the following paper) showing that the spin–lattice relaxation time is longer than 10^{-9} second.

The four remaining ions ($3d^1Ti^{+++}$, $3d^2V^{+++}$, $3d^6Fe^{++}$, $3d^7Co^{++}$) have a triplet level lowest, which will be split up in a field of low symmetry, but usually with a smaller separation than the ions considered previously. One may therefore expect a short spin–lattice relaxation time (Teunissen and Gorter 1939); in fact it has been found necessary to go to helium temperatures with Ti^{+++}, to hydrogen temperatures with Fe^{++}, and to oxygen temperatures with Co^{++}, to obtain resonance lines (no V^{+++} salt has been investigated).

In considering the question of the initial splitting of the spin levels it is not possible to draw such broad conclusions as in the last paragraph, except as regards the ions with Kramers' degeneracy. In all of these paramagnetic resonance has been detected at 10 cm. wavelength, using low temperatures where necessary. Of the other ions, one may mention Ni^{++} as a typical case, in most of whose salts no resonance has been detected at 3 cm. wavelength; this points to a rather large splitting (>1 cm^{-1}) as expected from previous work by other methods. Detailed consideration of the initial splitting is postponed, however, to the discussion of the individual ions in the following paper.

§ 6. CONCLUSION

It thus appears that all paramagnetic salts of the iron group should be open to investigation by the method of paramagnetic resonance, and one may note, in parenthesis, that the general considerations of this paper pertain also to salts of the rare earth group. The main difference lies in the fact that in the latter the crystalline field is not strong enough to destroy the coupling of the orbital and spin momenta to form a resultant **J**, but does partially lift the degeneracy of its $2J+1$ orientations. The splittings are much larger than those of the spin multiplets in the iron group, and lie more in the far infra-red than at centimetre wavelengths. Modulation of the crystalline electric field by the Debye waves will now affect the paramagnetic ion directly and one may expect that the lattice relaxation times will be short, except at low temperatures. Few data are available on this point except in the case of gadolinium, to which, being in an ^8S state, these considerations do not apply.

In conclusion, one may say that paramagnetic resonance offers a method of detailed investigation of paramagnetic ions which possesses the following advantages over previous methods: (i) no corrections for diamagnetism or diamagnetic impurities; (ii) direct measurement of effective g values; (iii) direct measurement of small splittings of the order of 1 cm^{-1}; (iv) direct observation of the different paramagnetic ions in unit cell; (v) observation of very short spin–lattice relaxation times of the order of 10^{-10} second.

In addition it is to be expected that the interpretation of the spectra will yield valuable information concerning the crystalline electric field, and the shapes and widths of the lines will form a test of the theories of spin–spin interaction.

Full utilization of the method of paramagnetic resonance for investigating the behaviour of the spin energy levels under the combined influence of the crystalline electric field and an external magnetic field requires measurements on single crystals at various orientations, wavelengths and temperatures. Such measurements are being made on a number of salts, principally copper sulphate, the alums and the Tutton salts, and will be reported fully elsewhere. The experimental work presented in the following paper has been concerned mainly with powders, and measurements have been made of salts containing paramagnetic ions belonging to each of the nine spectroscopic states possible in the iron group (except $3d^2$), in order to make a preliminary survey to test the main conclusions of this paper.

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Paramagnetic Resonance in Salts of the Iron Group—A Preliminary Survey: II. Experimental Results *

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ABSTRACT. Over a hundred salts of the iron group have been examined by the method of paramagnetic resonance at a wavelength of 3 cm., using magnetic fields up to 15 kilogauss at room temperature. The cases in which no absorption is observed have been further investigated at lower temperatures and shorter wavelengths. The results are considered for each paramagnetic ion in relation to the effects of the crystalline field theory. The ions Ti^{+++}, Fe^{++} and Co^{++} are found to have abnormally small spin–lattice relaxation times, while in Fe^{++} and Ni^{++} the energy levels are separated in zero magnetic field by as much as 1 cm^{-1} in most salts. A preliminary investigation of salts of the rare earth group indicates that spin–lattice relaxation times for most of these ions are also very short at temperatures above those of liquid hydrogen.

§ 1. INTRODUCTION

In the preceding paper it has been shown that it should be possible to detect resonance in all paramagnetic salts of the iron group, but that the effect may not be manifest at all wavelengths and temperatures. It should be possible to ascribe any failure to two causes—too great an initial splitting of the energy levels, or too broad an absorption line due to a very short spin–lattice relaxation time. In the hydrated ions both these effects are closely related to the splitting of the orbital levels by the crystalline field, which is determined by the spectroscopic state of the ion, and by the arrangement of its surrounding molecules in the crystal. In spite of considerable variations in the latter, the crystalline electric field has in general predominantly cubic symmetry, and the success or failure in detecting paramagnetic resonance should be primarily associated with the spectroscopic state of the ion.

To test these conclusions, a preliminary survey of some hundred salts was made at room temperature, using a wavelength of three centimetres and magnetic fields up to 15 kilogauss. It was found that about forty salts failed to show any effect, and most of these have been further investigated at liquid oxygen and liquid hydrogen temperatures and at shorter wavelengths. In this paper the results of these measurements are presented and analysed, ion by ion, in terms of the general discussion of the preceding paper.

§ 2. THE EXPERIMENTAL METHOD

In this section a brief outline of the essentials of the experimental method will be attempted, without any detailed discussion of the technique.

The paramagnetic resonance line is detected in absorption by the damping of a cavity resonator. At the shorter wavelengths this consists generally of a section of circular waveguide excited in the H_1 mode, the crystal being placed at one end in a position of maximum magnetic field. At longer wavelengths either a rectangular

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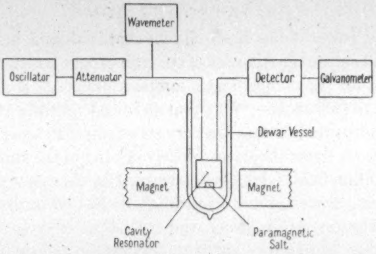
* This work was discussed at the Physical Society Conference at Oxford on 23rd–24th July 1948

waveguide resonator is used, excited again in the lowest H mode, or a quarter-wavelength section of coaxial line open-circuited at one end and close-circuited at the other. An external magnetic field is applied perpendicularly to the radio-frequency magnetic field.

The magnification factor Q of the empty cavity is normally several thousand, and Kittel and Luttinger (1948) have shown that in an average case this would be reduced to a few hundred at the centre of a paramagnetic absorption line, if the cavity is filled with salt. This arrangement is impracticable for measurements on single crystals and, moreover, most of the salts to be investigated show strong dielectric absorption at centimetre wavelengths. This dielectric absorption is minimized by placing the crystal at a node in the electric field, but owing to its finite thickness the crystal will project slightly into the electric field. The amplitude of the electric field increases sinusoidally with distance from the end, and it is easy to show that the optimum thickness of crystal (i.e. that producing maximum paramagnetic absorption in proportion to other losses in the resonator) is that whose dielectric losses reduce the Q to one-third of the value for the empty cavity. Under these conditions the reduction in the Q due to the paramagnetic resonance is relatively small, and is measured by the change in the power transmitted through the cavity from an oscillator to which it is tuned (see figure).



Block diagram of apparatus.

This power is measured by a silicon-tungsten crystal rectifier which, when operated at low power, gives a rectified current through a low resistance galvanometer closely proportional to the R.F. power. Then the paramagnetic absorption can be calculated from the formula

$$\text{Fractional increase in } 1/Q = (\delta_0/\delta_1)^{\frac{1}{2}} - 1, \qquad \ldots\ldots(1)$$

where δ_1, δ_0 are the galvanometer deflections in the presence and absence of paramagnetic resonance absorption respectively.

The method of measurement may be summarized as follows. A crystal of the paramagnetic salt is placed in the cavity, which is tuned to an oscillator of the desired frequency. An external magnetic field is then applied, and slowly varied in magnitude until the resonance is observed. The absorption curve due to the salt is then delineated by plotting the quantity given in equation (1) as a function of the external field. At each setting of the field the tuning either of the cavity or the oscillator is adjusted to give maximum galvanometer deflection, to avoid errors due to drift of the oscillator in frequency, to detuning of the cavity by the anomalous dispersion in the paramagnetic salt, or to distortion caused by magnetic forces.

The experimental technique at low temperatures is fundamentally the same, though rather more complicated than at room temperature; this drawback is outweighed in many cases by a number of advantages. Apart from the necessity of using low temperatures when the spin–lattice relaxation time is too short at room temperature, the advantages are, in order of importance: (*a*) the absorptions are much stronger, corresponding to the increase in susceptibility according to Curie's law; (*b*) the damping of the resonator due to dielectric losses in the crystal is usually, though not invariably, negligible; (*c*) the *Q* of the empty cavity is somewhat higher.

Each of these effects enhances the absorption due to the paramagnetic resonance in comparison with other losses; this is especially valuable when it is difficult to grow large single crystals, and in studying the shapes of resonance lines in the wings.

§ 3. RESULTS

The great majority of the salts were measured first at room temperature at a wavelength of 3 cm., using magnetic fields up to 15 kilogauss. It is convenient to present the results of this work first, and then to discuss the success or failure to detect resonance in the light of further experiments (§ 4). Table 1 contains therefore a list of the salts investigated, grouped according to the spectroscopic state of the paramagnetic ion, with a dividing line to separate those salts in which no absorption due to paramagnetic resonance could be detected under the conditions: temperature 290° K., wavelength 3·2 cm., magnetic field, varied from zero to 15 kilogauss (see p. 554).

Before proceeding further it is convenient to eliminate from the list of failures those which may be ascribed to a reason trivial from the theoretical standpoint, namely, that the dielectric losses in the crystals masked the paramagnetic absorption. These losses were particularly great in the hygroscopic salts, owing to the enormous absorption in water at centimetre wavelengths. The latter disappears when the water is frozen, and, in general, the dielectric losses in the crystals themselves decrease markedly at low temperatures. Salts in which the failure to detect paramagnetic resonance is ascribed to this effect are marked with an asterisk, those salts in which the resonance appears at lower temperature owing to the decrease in dielectric losses are marked with a double asterisk. It should be noted also that susceptibility measurements indicate that the cobalt-ammine complexes are diamagnetic.

Of the salts in which the failure to observe resonance could not be ascribed to dielectric loss, a considerable number have been further investigated at shorter wavelengths and lower temperatures. The results are summarized in table 2, where the conditions under which resonance was detected are given at the foot of each column (see p. 555).

§ 4. REVIEW OF THE INDIVIDUAL IONS

A brief general review of the effects of the crystalline field on the orbital levels of the ions of the first transition group has been given in § 5 of the preceding paper. In the light of this review, the results just presented merit some further discussion, which is most conveniently assembled separately for each ion. As before, the sign of the coefficient D of the fourth order portion of the crystalline field potential will be taken to be positive, except where otherwise stated. The magnitude and sign of the coefficient λ of the spin–orbit coupling play an important rôle and are taken from the calculations of Laporte (1928).

Table 1. Salts investigated at 3 cm. wavelength, room temperature

Resonance observed				
$^4F_{3/2}$	$^5D_0$	^6S	$^3F_4$	$^2D_{5/2}$
VSO_4	$CrC_2O_4 \cdot H_2O$	MnF_2	$NiCl_2$	CuF_2
CrF_3	$CrCO_3$	$MnCl_2 \cdot 4H_2O$	$NiBr_2$	$CuCl_2 \cdot 2H_2O$
$CrCl_3$		$MnSO_4 \cdot 4H_2O$		$CuSO_4$
$CrCl_3 \cdot 6H_2O$		$K_2Mn(SO_4)_2 \cdot 6H_2O$		$CuSO_4 \cdot 5H_2O$
$Cr_2(SO_4)_3 \cdot 5H_2O$		$Mn(NH_4)_2(SO_4)_2 \cdot 6H_2O$		$Cu(NH_3)_4 \cdot SO_4 \cdot H_2O$
$Cr_2(SO_4)_3 \cdot 15H_2O$		$Mn(C_2H_3O_2)_2 \cdot 4H_2O$		$Cu(NH_3)_4Cl_2 \cdot H_2O$
$KCr(SO_4)_2 \cdot 12H_2O$		$Mn_3(AsO_4)_2$		$CuCl_2 \cdot 2NH_4Cl \cdot 2H_2O$
$NH_4Cr(SO_4)_2 \cdot 12H_2O$		$Mn_3(C_6H_5O_7)_2$		$CuK_2(SO_4)_2 \cdot 6H_2O$
$RbCr(SO_4)_2 \cdot 12H_2O$		$Mn_2Fe(CN)_6 \cdot 7H_2O$		$CuCs_2(SO_4)_2 \cdot 6H_2O$
$(NH_3CH_3)Cr(SO_4)_2 \cdot 12H_2O$		$Mn_3(PO_4)_2 \cdot 7H_2O$		$Cu(NH_4)_2(SO_4)_2 \cdot 6H_2O$
$Cr(NO_3)_3 \cdot 9H_2O$		$Mn(CHO_2)_2 \cdot 2H_2O$		
$CrPO_4 \cdot 3H_2O$		$Mn_2(P_2O_7) \cdot 3H_2O$		
$Cr(C_2H_3O_2)_3 \cdot H_2O$		$(MnC_2O_4)_2 \cdot 5H_2O$		
$Cr(OH)_3$		$MnCO_3$		
$Cr_2(CrO_4)_3$		$Mn(H_2PO_2)_2 \cdot H_2O$		
$CsCr(SO_4)_2 \cdot 12H_2O$		$Mn(C_3H_5O_3)_2 \cdot 3H_2O$		
		$(FeF_3)_2 \cdot 9H_2O$		
		$FeCl_3$		
		$Fe_2(SO_4)_3 \cdot 9H_2O$		
		$FeNH_4(SO_4)_2 \cdot 12H_2O$		
		$FeOH(C_2H_3O_2)_2$		
		$Fe(C_7H_5O_2)_3$		
		$Fe_4(Fe(CN)_6)_3$		
		$Fe(PO_4) \cdot 4H_2O$		
		$Fe_2[C_3H_5(OH)_2OPO_3]_3$		
		$Fe_2(C_2O_4)_3$		

No resonance observed					
$^2D_{3/2}$	^6S	$^5D_4$	$^4F_{9/2}$	$^3F_4$	$^2D_{5/2}$
$CsTi(SO_4)_2 \cdot 12H_2O$	$MnBr_2$**	$FeCl_2$	$CoCl_2 \cdot 6H_2O$	NiF_2	$CuBr_2$**
	MnI_2**	$FeI_2 \cdot 4H_2O$	$CoSO_4 \cdot 7H_2O$	$NiCl_2 \cdot 6H_2O$	$Cu_3(PO_4)_2 \cdot 3H_2O$
	$MnNH_4PO_4 \cdot H_2O$	$FeSO_4 \cdot 7H_2O$	$Co(NO_3)_2 \cdot 6H_2O$	NiI_2	
	$FeCl_2 \cdot 6H_2O$*	$Fe(NH_4)_2(SO_4)_2 \cdot 6H_2O$	$Co(NH_4)_2(SO_4)_2 \cdot 6H_2O$	$NiSO_4 \cdot 7H_2O$	
	$Fe(CHO_2)_3 \cdot H_2O$	$K_2SO_4FeSO_4 \cdot 6H_2O$	$NH_4ClCoCl_2$	$K_2Ni(SO_4)_2 \cdot 6H_2O$	
	$K_3Fe(CN)_6$	$(FeSO_3)_2 \cdot 5H_2O$	$Co(H_2O)(NH_3)_5Cl_3$	$Ni(NO_3)_2 \cdot 6H_2O$	
		$Fe_3(PO_4)_2 \cdot 8H_2O$	$[Co(NH_3)_5Cl]Cl_2$	$Ni_3(PO_4)_2 \cdot 7H_2O$	
		$FeC_2O_4 \cdot 2H_2O$	$[Co(NH_3)_4(CO_3)]NO_3 \cdot 6H_2O$	$Ni(CHO_2)_2 \cdot 2H_2O$	
		$FeBr_2 \cdot 6H_2O$	Cs_3CoCl_5	$Ni(NH_4)_2(SO_4)_2 \cdot 6H_2O$	
		$Co_2 \cdot O_3$			

Headings of columns indicate spectroscopic state. For meaning of asterisks see text.

Table 2. Substances further investigated at lower temperatures and shorter wavelengths

			Resonance observed		
	Ti^{+++}	$Fe^{+++}Mn^{++}$	Fe^{++}	Co^{++}	Ni^{++}
	$CsTi(SO_4)_2$ $12H_2O$	$K_3Fe(CN)_6$	$FeK_2(SO_4)_2.6H_2O$ $Fe(NH_4)_2(SO_4)_2.6H_2O$	$CoCl_2.6H_2O$ $Co(NO_3)_2.6H_2O$ $CoSO_4.7H_2O$ $CoK_2(SO_4)_2.6H_2O$ $Co(NH_4)_2(SO_4)_2.6H_2O$ Cs_3CoCl_5	$NiCl_2.6H_2O$ $Ni(NH_4)_2(SO_4)_2.6H_2O$ $NiSiF_6.6H_2O$
T (° K.)	6	20	20	90	90, 290
λ (cm.)	10	3	1·3	3	1·3, 0·8
			No resonance observed		
		Mn^{++}		Co^{++}	Ni^{++}
		$Mn(NH_4)PO_4.H_2O$		$Co(H_2O)(NH_3)_5Cl_3$ NH_4CoCl_3	NiF_2 NiI_2 $NiSO_4.7H_2O$ $Ni(NO_3)_2.6H_2O$ $NiK_2(SO_4)_2.6H_2O$
T (° K.)		90		90	90
λ (cm.)		3		3	1·3

T = temperature. λ = wavelength.

Headings of columns indicate spectroscopic state.

$3d^1$, ${}^2D_{3/2}Ti^{+++}$, $\lambda = 154\,cm^{-1}$

The five-fold orbital level is split in a cubic electric field into a triplet and a doublet, the former being the lower when the sign of the coefficient *D* is positive. The degeneracy of the triplet is lifted by a rhombic field, but the exact splitting is not known. The spin is $\frac{1}{2}$, and no electric field can separate the two spin levels by Kramers' (1930) theorem. The absence of paramagnetic resonance in caesium titanium alum must therefore be due to a very short spin–lattice relaxation time; this is consistent with negative results of relaxation experiments (Gorter, Teunissen and Dijkstra 1938, de Haas and du Pré 1938). No resonance was detected at 20° K. and 3 cm. wavelength, but Bijl (unpublished) has since found in this laboratory that it appears at 6° K., using a wavelength of 10 cm. Such a short relaxation time could be explained by the theory of Van Vleck (1930), if the orbital splitting were as low as 100 cm^{-1}, the dominant effect being the " Raman process ". The effective *g* value is considerably less than two, as would be expected from the positive spin–orbit coupling and the small orbital splitting.

$3d^2$, ${}^3F_2V^{+++}$

In a cubic field the seven-fold orbital level splits into a singlet and two triplets, with a triplet lowest, which will be split in a field of lower symmetry (see Siegert 1936, 1937). The spin triplet is split by a trigonal field into a singlet ($M_S = 0$) and a doublet ($M_S = \pm 1$), with a separation of 5 cm^{-1}, according to susceptibility measurements on vanadium ammonium alum of van den Handel and Siegert (1937) (see also Van Vleck 1939 a). Except for weak transitions between +1 and −1 levels, one would not expect paramagnetic resonance to be detectable except at low temperatures and millimetre wavelengths. No measurements have yet been made.

$3d^3$, ${}^4F_{3/2}Cr^{+++}$, $\lambda = 87\,cm^{-1}$; V^{++}, $\lambda = 55\,cm^{-1}$

In a cubic field the orbital singlet term lies lowest, some 10^4 cm^{-1} below the nearest triplet. The spin quadruplet remains completely degenerate in a cubic field, but is resolved into two doublets by any field of lower symmetry, the Kramers degeneracy always being present. Thus all Cr^{+++} and V^{++} salts should show paramagnetic resonance at room temperature at all wavelengths. The separation of the two spin doublets is generally rather small (a fraction of 1 cm^{-1}), and magnetically dilute Cr^{+++} salts such as the alums have been widely used for adiabatic demagnetization experiments below 1° K. Some measurements at room temperature by the method of paramagnetic resonance have already been reported by Bagguley and Griffiths (1947) and by Weiss, Whitmer, Torrey and Hsiang (1947), who find a separation of 0·12 cm^{-1} for the potassium alum and 0·15 cm^{-1} for the ammonium alum. Bleaney and Penrose (1948 b) have found that as the salts are cooled down the splittings change in a marked fashion. Complete reports of measurements, both at room temperature and at low temperatures, on these and three other chrome alums will be presented shortly.

The abnormally narrow line obtained with $CrCl_3$ has already been commented on in a footnote of the preceding paper. The fact that only a single line is obtained is probably due to exchange forces strong enough to overcome the effects of the crystalline electric field.

Only one divalent vanadium salt has been examined: the sulphate. This gave a fairly narrow absorption line close to the field for $g = 2$.

$3d^4$, $^5D_0Cr^{++}$, $\lambda = 57\,cm^{-1}$

In a cubic field the orbital levels split, like those of copper into a lower "non-magnetic" doublet and an upper triplet. As regards the perturbation of the spins, only the rather large energy separations between the lower level of the doublet and the triplet levels play any part, since the orbital doublet is non-magnetic. Also, the spin–orbit coupling coefficient is rather small, and one may expect the initial splittings of the spin quintuplet to be small, and the spin–lattice relaxation time to be fairly long. These predictions are confirmed by the observation of paramagnetic resonance at room temperature in the salts investigated.

$3d^5$, $^6S_{5/2}Fe^{+++}$, Mn^{++}

These ions are in S states when free, and it follows that effects of the crystalline electric field are very small indeed. The spin sextuplet is resolved into a doublet and a quadruplet by a cubic field, and into three doublets by a trigonal field. As expected, resonance was detected in practically all the salts examined. A special case worthy of note is potassium ferricyanide, where the bonding is partly covalent and the much stronger electric fields thereby produced are able to break down the Russell-Saunders coupling (Howard 1935). The spins pair off as far as possible, making the ferrocyanide diamagnetic, while in the ferricyanide the susceptibility approaches that of a single spin. In an experiment at 20° K. and 3 cm. wavelength an exceedingly broad absorption was observed, increasing with field, but the maximum was not reached at 8000 gauss.

Single crystals have been investigated of two salts used in low temperature adiabatic demagnetization work: iron ammonium alum and manganese ammonium sulphate. Owing to the large spin the line breadth is so great that accurate measurements of splittings are impossible. In the alum the splitting appears to be not more than $0{\cdot}10\,cm^{-1}$; this conflicts with low temperature measurements of Benzie and Cooke (1948), which give $0{\cdot}14\,cm^{-1}$, assuming a cubic field. The explanation may lie either in a trigonal field which decomposes the energy levels into three doublets of small separation, or in a magnetic specific heat larger than that calculated from dipole–dipole interaction, which would correspondingly reduce the specific heat attributed to the Stark splitting and hence the estimate of the splitting from low temperature measurements. In the manganese Tutton salt the experiments suggest that the energy levels are split into three doublets by the crystalline electric field, with separations of the order of 0·1 and $0{\cdot}2\,cm^{-1}$.

$3d^6$, $^5D_4Fe^{++}$, $\lambda = -100\,cm^{-1}$

In a cubic field the orbital levels split to a basic triplet and an upper doublet, being fully resolved by a rhombic field. The latter will also fully resolve the spin quintuplet, though fields of higher symmetry leave some degeneracy. There appear to be practically no data from which an estimate of the splittings can be made but one may expect them to be fairly large, and the spin–lattice relaxation time to be short. No resonance has so far been detected at 3 cm. using fields up to 8 kilogauss at temperatures down to 90° K., though a weak absorption appears at 20° K. Measurements at 1·3 cm. wavelength have revealed strong resonance lines in ferrous ammonium and ferrous potassium sulphate at 20° K., though nothing could be detected at 90° K. The potassium salt showed strong absorption in zero field, indicating that one of the splittings of the spin quintuplet must be about $0{\cdot}8\,cm^{-1}$ in this case.

$3d^7$, $^4F_{9/2}Co^{++}$, $\lambda = -180\,cm^{-1}$

The orbital levels are split up in the same way as those of V^{+++}, but the spin levels can only be resolved into two doublets by an electric field. Their separation will be much greater than that in Cr^{+++}, owing to the larger value of λ and the smaller orbital splitting, and Van Vleck (1939 b) has suggested that the splitting may be as much as $10\,cm^{-1}$. It should be possible, however, to detect paramagnetic resonance due to transitions between the $\pm\frac{1}{2}$ levels, and the negative results at room temperature must be due to a short spin–lattice relaxation time, as would be expected from the small orbital splitting. Measurements at lower temperatures have confirmed this, broad lines being obtained at 90° K. With single crystals of the Tutton salts, fairly narrow lines are observed at 20° K. with effective g values as high as three, and very considerable anisotropy. The spectrum is more complicated than would be expected with an effective spin of $\frac{1}{2}$, and the theoretical analysis may be rather involved (Schlapp and Penney 1932). The increased width of the lines at 90° K. suggests that the spin–lattice relaxation time is about 10^{-9} seconds at this temperature.

The cobalt salts shown in table 1 (except Cs_3CoCl_5) are all pink-coloured, showing that the cobalt ion has six coordination bonds, corresponding to a positive value of the coefficient D of the cubic field. In the blue-coloured salts the co-ordination number is four, and the sign of D should be reversed. A singlet orbital level should then be lowest, as in Cr^{+++} in a cubic field. These blue salts should then show paramagnetic resonance at room temperature, since the spin–lattice relaxation time should not be unduly short, and g values close to two. Krishnan and Mookherji (1938) have found that Cs_3CoCl_5, which Powell and Wells (1935) have shown to be four-coordinated, exhibits less anisotropy and its mean susceptibility is somewhat closer to the "spin-only" value than most cobalt salts. Investigation by means of paramagnetic resonance reveals no significant difference in behaviour however; it is necessary to cool the salt to 90° K. to obtain a narrow resonance curve, and the g value does not appear significantly smaller than in other cobalt salts.

$3d^8$, $^3F_4Ni^{++}$, $\lambda = -335\,cm^{-1}$

In a cubic field the orbital levels of Ni^{++} are similar to those of Cr^{++}, but inverted to those of Co^{++} and V^{+++}; the lowest level is therefore a singlet. The spin triplet is decomposed by a non-cubic field as in V^{+++}, but the splitting is smaller. Schlapp and Penney (1932) estimate its splitting to be $1{\cdot}5\,cm^{-1}$, but in fitting their calculations to the measurements of Bartlett on the Tutton salts they ignore the presence of two ions in unit cell and assume that the axes of the crystalline field are the same as those of the magnetic susceptibility. From specific heat measurements at low temperatures Giauque and Stout (1941) obtain an overall splitting of $5{\cdot}3\,cm^{-1}$ for $NiSO_4.7H_2O$, assuming three almost equally spaced levels. Broer, Dijkstra and Gorter (1943) point out that the measurements of Krishnan and Mookherji (1938) on anisotropy in this salt and $Ni(NH_4)_2(SO_4)_2.6H_2O$ lead to splittings two to three times larger than they obtain from paramagnetic relaxation: the latter gives $1{\cdot}5\,cm^{-1}$ for both salts (assuming a doublet and a singlet level). There is thus considerable uncertainty in the value of the splitting in Ni^{++} salts.

Measurements at a wavelength of 1·3 cm. on nickel ammonium sulphate have revealed a paramagnetic resonance line in rather high fields (about 12 kG.) though nothing could be detected in nickel potassium sulphate. This suggests that the splitting in the former salt is not much greater than 1 cm^{-1}, resonance being detected at a lower frequency (0·8 cm^{-1}) because one of the $M_S = \pm 1$ energy levels may converge in a magnetic field on the $M_S = 0$ level. No resonance was observed at 3 cm. at temperatures down to 20° K., though it is possible that a line may appear if a sufficiently large magnetic field could be applied to move the energy levels still closer. Evidently the transition between the $+1$ and -1 levels, assuming them to be degenerate in zero magnetic field is very weak, since it is not detectable at 20° K.

The detection of paramagnetic resonance in two unhydrated nickel halide salts shows that either the crystalline field is too small to produce a splitting of 0·3 cm^{-1}, or that exchange forces arise powerful enough to overcome the effects of the crystalline field. ($NiCl_2$ has a Curie point at 50° K.). No resonance lines could be detected in the anhydrous fluoride or iodide at either 3 cm. or 1·3 cm. wavelength, even at 90° K.

A hydrated salt in which the splitting of the triplet is small is $NiSiF_6.6H_2O$. From measurements of paramagnetic rotation of the plane of polarization of light by Becquerel and van den Handel (1939) down to liquid helium temperatures, a splitting of 0·3 cm^{-1} was deduced by Becquerel and Opechowski (1939). Since this crystal contains only one ion per unit cell, it is particularly simple for study by the paramagnetic resonance method. Measurements by one of us (R.P.P.) have shown that the splitting is 0·32 cm^{-1} at 195° K., decreasing as the temperature is lowered to 0·17 cm^{-1} at 90° K. and 0·12 cm^{-1} at 20° K.; no further change occurred down to 14° K. Since the splitting is entirely due to the non-cubic part of the crystalline field, the change is probably due to anisotropic thermal contraction, which should vanish near the absolute zero. The g value is 2·28.

$3d^9$, $^2D_{5/2}Cu^{++}$, $\lambda = -852\,cm^{-1}$

In a cubic field, the orbital levels split into a "non-magnetic" doublet and triplet, the former being the lower. Thus one may expect a fairly long spin–lattice relaxation time, and in view of the Kramers' degeneracy practically all hydrated copper salts should show paramagnetic resonance at room temperature at 3 cm. wavelength. The spin–orbit coupling coefficient is very large however, and the effective g values are appreciably greater than two, being accompanied by considerable magnetic anisotropy. Single crystals of copper sulphate and the copper Tutton salts have been investigated, and the results obtained with the latter are in good agreement with the theory of Polder (1942). In the case of $CuSO_4.5H_2O$, where the effects of exchange forces are pronounced, it is necessary to use millimetre wavelengths (i.e. magnetic fields >10 kG.) to separate the absorption lines associated with the two ions in unit cell (Bagguley and Griffiths 1948, Pryce 1948).

§ 5. DISCUSSION: LINE BREADTH

Though the presence of magnetic anisotropy can be detected by measurements of susceptibility on single crystals, its effects are more marked in paramagnetic resonance than in any other phenomenon. In nickel ammonium sulphate for instance, a strong resonance line was observed at 1·3 cm. wavelength in one orientation with respect to the magnetic field, whereas only a very slight absorption could be detected on turning the crystal through a right angle. Thus it was desirable as well as convenient to make most of the measurements in this preliminary survey on powders, in which all orientations are present simultaneously. The resultant absorption is then rather broad, consisting of many lines superimposed, and little can be said about the shape or width of the individual lines. In a number of cases observations have been made on single crystals, which have shown that the line shape and width depend very much on the orientation and on the crystal. Even in the cases where the width is ascribed to the effects of spin–spin interaction alone, no general expression for the shape can be given. Lines have been observed (in caesium chromium alum) whose shape conforms closely to a Gaussian distribution, as assumed by Broer (1943) in his theory of spin–spin relaxation, and also (in $CuSO_4.5H_2O$) to the simple Lorentz pressure-broadening shape for spectral lines in gases; but these are exceptional cases, and a variety of intermediate shapes have been found. The complexity of the problem is illustrated by the copper Tutton salts, where the maximum and minimum widths for different orientations of the crystal (themselves different by a factor three) vary by a factor three between salts with different monovalent metallic ions.

In salts where spin–lattice relaxation is the dominant effect, it is reasonable to assume that the line shape (as a function of frequency) should be given by an expression of the form

$$F(\nu)=\left\{\frac{\Delta\nu}{\Delta\nu^2+(\nu-\nu_0)^2}+\frac{\Delta\nu}{\Delta\nu^2+(\nu+\nu_0)^2}\right\},$$

where $\Delta\nu=1/\rho_L$, since the assumption that the time between "collisions" with the lattice during which an ion can absorb energy from the radiation field has an exponential distribution of the type $f(t)=\exp(-2\pi t/\rho_L)$ is the same as in the theory of Van Vleck and Weisskopf (1945) and Fröhlich (1946) for pressure broadening. If paramagnetic resonance lines could be observed at constant magnetic field (i.e. constant ν_0) and the absorption curve delineated as a function of frequency, it would be a simple matter to deduce the relaxation time. As they are in fact observed at constant frequency and at variable field, it is difficult to obtain more than an estimate of the relaxation time from a single measurement since in general ρ_L is a function of the external magnetic field, and the line shape is obtained as a function of the field and not of the frequency. By making measurements at different frequencies it should be possible to interpret the curves, however, and obtain the spin–lattice relaxation time at different temperatures and fields.

In the preceding paper it was suggested that the ions of the rare earth group (except Gd^{+++}) might have very short spin–lattice relaxation times. Some preliminary experiments to test this were made with a few salts kindly lent to us by Mr. J. R. Marsh of the Inorganic Chemistry Laboratory, Oxford. At room temperature and 3 cm. wavelength only gadolinium (bromate and acetate), terbium (bromate) and europium (chloride) showed any absorption. A further investigation was therefore made at low temperatures, using a wavelength of 10·4 cm. It was found that all the ions examined (cerium, neodymium, samarium, europium, terbium, dysprosium, erbium and ytterbium), showed resonance lines at 20° K., but nothing could be detected at 90° K. except in europium. In general, then, the spin–lattice relaxation time is very short at high temperatures, as expected from the strong coupling between spin and orbit. The case of the europium salt, $EuCl_3.6H_2O$, is very puzzling, as the absorption was considerably stronger at 20° K. than at 90° K., whereas all the ions should be in the lowest non-magnetic state ($J=0$) at this temperature.

When the spin–lattice relaxation time is long compared with the spin–spin relaxation time, its contribution to the widths of the lines is negligible. Nevertheless if sufficient power can be provided to develop the saturation phenomenon, as found in the nuclear magnetic resonance experiments (see, for example, Bloembergen, Purcell and Pound 1948) and in the inversion spectrum of ammonia

(Bleaney and Penrose 1948 a) it should be possible to measure long spin–lattice relaxation times, either by applying short pulses and observing the variation of the paramagnetic absorption with time on an oscilloscope or by measuring the diminution in the equilibrium value of the absorption as the radio-frequency power level is increased. At low temperatures, where the spin–lattice relaxation time increases rapidly, the microwave power required is quite small, of the order of milliwatts when $\rho_L > 10^{-3}$ second. It is difficult at this stage to estimate the merits of this type of measurement in comparison with the conventional methods of paramagnetic relaxation.

§6. CONCLUSION

The chief merits of the method of paramagnetic resonance for the investigation of the energy levels of paramagnetic ions have been summarized at the end of the preceding paper, where it was pointed out that measurements on single crystals are essential to make full use of the method. A few such measurements have been mentioned in this paper, but the bulk of the observations have been made on powders in order to obtain a fairly rapid survey of the iron group. The results indicate that the main features of the resonance phenomenon are consistent with the crystalline field theory in its broad outline. More detailed measurements will show how far the finer points of the theory are supported by experiment, and should throw light on the nature of the spin–spin interaction.

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Atmospheric Absorption of Millimetre Waves *

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ABSTRACT. The attenuation of electromagnetic waves in the region of 5 mm. wavelength due to resonance in the oxygen molecule was measured over atmospheric paths up to 2 km. long, using a fixed transmitter and a mobile superheterodyne receiver. For ten wavelengths between 6·34 and 4·48 mm. values were obtained which, after reduction to standard dry atmospheric conditions, range up to a maximum of 15·7 db/km. at the peak of the absorption curve. They agree substantially with the theory of Van Vleck, and give a value of line-breadth constant $\Delta\bar{\nu}$ of about 0·02 cm^{-1}, confirming estimates derived from other methods.

§ 1. INTRODUCTION

THE existence of a molecular resonance in oxygen giving rise to absorption of electromagnetic radiation in a wavelength region around 5 mm. has been predicted on theoretical grounds by Van Vleck (1947 a), who has discussed the theory at length. The absorption arises from the permanent magnetic moment associated with the oxygen molecule, transitions between closely-spaced components in the ground state giving resonances in the millimetre wave region. At atmospheric pressures the effect of collision broadening reduces individual resonances to a single broad band. An undetermined constant in the theory was the line-breadth constant $\Delta\bar{\nu}$, the wave-number half-width at half-intensity, which originally could only be estimated roughly.

Laboratory measurements confirming the effect have been made by Beringer (1946), who used a short length of waveguide filled with pure oxygen or a mixture of oxygen and nitrogen, and measured the attenuation constant at various wavelengths. His results gave a value for $\Delta\bar{\nu}$ between the limits 0·02 and 0·05 cm^{-1}.

The present paper describes experiments already briefly reported (Lamont 1948) carried out in the open air to measure the attenuation rate as it actually occurs in the atmosphere. The measurements were done on an R.A.F. aerodrome during the summer of 1947. Wavelengths in the region between 6·34 mm. and 4·48 mm. were used, with distances between transmitter and receiver up to 2·2 km. As will appear, the results provide satisfactory confirmation of the theory, and add weight to previous estimates of the line-breadth constant.

§ 2. APPARATUS

The equipment used was built for a general investigation of the propagation characteristics of millimetre waves, and some early work with it on propagation over sea has been reported (Lamont and Watson 1946). Briefly, the transmitter consists of a velocity-modulation oscillator feeding into a distorter crystal, in the form of a germanium crystal valve mounted in a cylindrical resonator, from which second harmonic power is extracted. This is mounted behind a 16-inch diameter aluminium paraboloidal reflector, which is fed from the front through a waveguide and small flare, the radiation being vertically polarized. Square-wave modulation at 1000 c/s. repetition rate is applied to the cathode screen.

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* Communication from the Staff of the Research Laboratories of The General Electric Company, Limited, Wembley, England.

The receiver is of superheterodyne type, with a germanium crystal mixer, and a velocity-modulation local oscillator operating at half the signal frequency. The aerial system is similar to that of the transmitter. A miniature I.F. amplifier of about 10 Mc/s. bandwidth is mounted directly below the mixer, and is connected by cable to an L.F. amplifier having a cathode-ray oscillograph and rectifier meter connected to its output. The receiver is remotely tuned by a Selsyn drive. The wavelength range was covered by three different valve types, two of which were made by E.M.I. Ltd. specially for this work.

Since the beam width of the aerial is small (about 1° between half-power points) each aerial carries a telescope aligned with the beam, and the aerials are then directed entirely by this optical means.

For these measurements the transmitter was on a fixed site, and the receiver was mobile, being mounted on a turntable on the roof of a small trailer, inside which was the measuring and calibrating equipment. Power was provided by a petrol-electric set in the towing vehicle, and the necessary voltage stabilization was incorporated in the power supply units.

§ 3. METHOD OF MEASUREMENT

If the transmitter and receiver are separated by a distance d, and the received signal is assumed to be free from ground reflections or other spurious echoes, then the received electric field E will be determined by the usual $1/d$ law with an additional attenuation α due to atmospheric absorption, so that $E=(E_0/d)e^{-\alpha d}$. From relative measurements of E made at a number of known distances a value of α can be determined.

For calibration of the receiver use is made of the well-established property that the I.F. output of a mixer crystal is proportional to the H.F. signal, so long as these are small compared with the amplitude of the local oscillator swing. A signal at 60 Mc/s., the mid-band frequency of the I.F. amplifier, is fed from a signal generator through resistance decoupling to the output side of the crystal mixer. This signal is modulated with a square wave of the same characteristics as the transmitter modulation, and is observed at the output of the receiver alternately with the signal received through the mixer. The change-over is done by means of two relays, one of which is a type with high-frequency contacts which form a switch in the coaxial cable from signal generator to mixer. The other relay is mounted at the side of the aerial mirror, and, through a push rod, operates two spring fingers which open or close over the end of the waveguide feed to the mirror. Thus operation of a foot switch effects instant change-over from received to calibrating signal, and the signal generator output can be rapidly adjusted to match the received signal in amplitude. Spot calibration of this type is necessary in order to make the measurements sufficiently independent of variations in the I.F. and L.F. gains over the comparatively long period required for a set of readings. The supply voltage and the output monitor reading of the signal generator were reset if necessary to their prescribed values during each reading.

Thus readings are dependent on the attenuator of the signal generator, and this has been calibrated by the National Physical Laboratory at the operating frequency to an overall accuracy of ±0·1 db. The accuracy of setting of the output monitor and the accuracy of matching of signals are both better than 0·1 db.

The transmitter is not monitored, other than by noting values of the rectified current in the distorter crystal. This is not in itself a good indication of power output, since the efficiency of frequency doubling is poor, and bears little relation to the driving power, but it does provide a check lest any variation in crystal characteristic should occur. The harmonic power output is of the order of 10 μw., for which level no satisfactory means of monitoring has yet been found. The frequency is determined by means of a cavity wavemeter loosely coupled to the waveguide connecting oscillator and distorter cavity, the accuracy being to within about 1 part in 1000.

§ 4. EXPERIMENTS

Measurements based on the relation quoted above assume free-space propagation conditions. An attempt was made first of all to find a site where such conditions would be evidently fulfilled, for example a deep valley with a road running along the top on one side, but nothing practicable was found.

The next best scheme was to use a flat piece of land with a tower on which the transmitter could be mounted. With this arrangement it will be seen from figure 1 that, although the receiver height may be small, the angle between direct and reflected rays at the receiver is dependent only on the transmitter height, and, therefore, with a sufficient transmitter height, a receiving aerial of narrow beam angle will discriminate between these rays.

A suitable site was found on an R.A.F. aerodrome, where permission was obtained to mount the transmitter on the flat roof of the Flying Control building and to use the perimeter tracks and runways for signal strength measurements. The transmitter height thus obtained was 46 feet.

After preliminary runs to try out the equipment, a number of changes were made, notably the addition of the rapid change-over from measured to calibrating signal described above, which improved both the speed and accuracy of the measurements. The tests included a check on the importance of differences between the modulation of transmitter and calibrating signal. It was found that the calibration was independent of wide changes in the repetition rate of one of the modulators, and was affected only in absolute level by changes in pulse width. Thus the two modulation waveforms must remain constant but need not be identical.

During the preliminary tests a number of sites were selected on the runways and perimeter tracks. They were all positions marked either by taxi lights or flare path lights, whose positions could be determined accurately from their representations on a map of the aerodrome having grid lines at 100-foot intervals. About 12 sites were used, varying in range from 0·121 km. to 2·193 km. The site nearest to the transmitter was the origin and terminal point of all runs, and the selection of sites for each run was made as a result of experience. It was usual to repeat all outward points on the inward run, but on occasions a circular tour was made.

If measured signals are to have their free-space value within ±0·1 db., which was the accuracy aimed at, the amplitude of an interfering signal caused by ground reflection must have a value of less than 1·2% of the direct signal, that is, it must be at least 38 db. below the direct signal. It was found from the aerial polar diagram that the gain is at least 20 db. below the maximum for all radiation received at angles greater than 1·4° from the axis. From the formula of figure 1, $\theta = 2h_1/d$ (h_1 being the transmitter height), θ will be greater than 1·4° for values of d less than 1·1 km. Thus ground reflections should give no trouble within this range if the remaining 18 db. at least is lost at the point of reflection, which corresponds to a reflection coefficient of about 0·1. No direct measurement of this reflection coefficient was attempted, but an estimate of its value can be obtained from the work of Ford and Oliver (1946) at a wavelength of 9 cm. Their measurements show that a covering of vegetation on level ground entirely eliminates specular reflection, and with vegetation about 40 cm. high a reflection coefficient of 0·04 was obtained with vertical polarization and an angle of incidence of 68°. Rough ground covered with long thick grass gave a value of 0·08 for the same conditions. Scaled to 5 mm. wavelength, these surfaces are probably rather smoother than the moderately short grass of the aerodrome, and, although the reflection coefficient will increase as grazing incidence is approached, it seems that a maximum value of 0·1 under the conditions of the present experiments is not unreasonable.

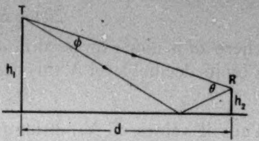

Figure 1. Angular relations between direct and reflected waves.
If $d \gg h_1$, then $\theta = 2h_1/d$, $\phi = 2h_2/d$.

The site at a range of 1·047 km. was the most distant site at which signals could be measured accurately for the wavelengths where the attenuation was high. For the lower attenuations the range was extended to a maximum of 2·193 km. These latter are beyond the range established above for which interference from ground reflections should be negligible, and in fact reflections were found to exist at the more distant sites. The interference was not regular, and, presumably on account of a complex interference field from a reflecting surface which in places was far from plane, sites differing in position by only a few yards sometimes gave quite markedly different signal levels. Most of these points have been included in the calculations since it is considered that the net effect of in-phase and out-of-phase reflections averaged over several sites will be small.

Ten different wavelengths were used, varying between 6·34 and 4·48 mm. The shortest wavelength was obtained, for the transmitter, as the third harmonic of a 13·44 mm. oscillation. In this case the beating oscillator of the receiver (kindly lent by the Clarendon Laboratory, Oxford) operated at 8·96 mm. wavelength. With the exception of the shortest wavelength, the experiments were carried out between 1st August and 5th September 1947, a period of fine warm weather. The experiments on the final wavelength were done during a cold spell at the end of November. The results are calculated from a total of thirty-five runs, each comprising, on the average, seven readings.

Values of air temperature, pressure and humidity were obtained from readings taken hourly by the meteorological staff of the aerodrome in the vicinity of the transmitter.

§ 5. RESULTS

The field strength at a distance d is given, as already stated, by

$$E = (E_0/d)e^{-\alpha d}. \qquad \ldots\ldots(1)$$

Measurements are obtained in the form of values of E expressed in decibels relative to an arbitrary level for known values of d. Putting the formula, therefore, in logarithmic form, we get

$$20 \log E + 20 \log d = 20 \log E_0 - \alpha d, \qquad \ldots\ldots(2)$$

where αd is now expressed in decibels instead of nepers as in (1). On replacing E by its decibel level N this becomes

$$N + 20 \log d = -\alpha d + N_0. \qquad \ldots\ldots(3)$$

Thus if values of $N + 20 \log d$ are plotted as ordinates y against d as abscissa the points should lie on a straight line whose slope is $-\alpha$. Figure 2 is a typical set of readings for a single run, showing their deviations from this linear relationship.

The best straight line through actual points subject to small errors is most accurately determined by the Method of Least Squares. With the values of d taken as exact, this gives for α, the most probable value of α (Birge 1932),

$$\bar{\alpha} = \frac{\Sigma y \Sigma d - s\Sigma yd}{s\Sigma d^2 - (\Sigma d)^2}, \qquad \ldots\ldots(4)$$

where s is the number of points, and summations extend over these points. To calculate the probable error of $\bar{\alpha}$ we must also find the mean value $\bar{N}_0$ of the constant N_0, which is given by

$$\bar{N}_0 = \frac{\Sigma d^2 \Sigma y - \Sigma d \Sigma yd}{s\Sigma d^2 - (\Sigma d)^2}. \qquad \ldots\ldots(5)$$

When values given by (4) and (5) are substituted in the s equations (3), a set of residuals $v_1, v_2, \ldots. v_s$ is obtained, from which the probable error in $\bar{\alpha}$ is derived as

$$\epsilon = 0{\cdot}67 \left[\frac{\Sigma v^2}{s-2} \, \frac{s}{s\Sigma d^2 - (\Sigma d)^2} \right]^{\frac{1}{2}}. \qquad \ldots\ldots(6)$$

The values of $\bar{\alpha} \pm \epsilon$ thus obtained are shown in the third column of the table.

Run No.	λ	$\bar{\alpha} \pm \epsilon$	$\bar{\alpha}_c$	$\bar{\alpha}_m$	P.E.
23	6·34	0·2 ±0·15	0·05	0·05	0·15
25	5·76	1·24±0·35	1·08	1·0	0·1
26	5·76	0·98±0·1	0·83		
27	5·76	1·37±0·2	1·24		
28	5·76	1·22±0·1	1·08		
29	5·76	1·48±0·2	1·33		
31	5·60	2·19±0·4	2·07	1·8	0·2
34	5·60	1·80±0·2	1·71		
35	5·28	9·94±0·3	9·9	10·2	0·1
36	5·28	9·90±0·1	10·3		
37	5·28	9·75±0·35	10·1		
39	5·28	9·42±0·35	9·7		
40	4·96	14·60±0·4	15·2	14·7	0·2
41	4·96	14·30±0·3	14·8		
42	4·96	14·17±0·1	14·6		
43	4·96	14·22±0·25	14·7		
44	5·13	15·49±0·4	15·7	15·7	0·2
45	5·13	15·71±0·3	15·9		
46	5·13	15·26±0·2	15·5		
47	5·13	15·44±0·5	15·9		
48	5·04	14·28±0·2	14·6	14·5	0·15
49	5·04	13·96±0·3	14·2		
50	5·04	14·50±0·3	14·7		
51	5·10	14·87±0·3	15·15	13·9	0·1
52	5·10	13·99±0·2	14·35		
53	5·10	14·26±0·4	14·42		
54	5·10	13·35±0·15	13·6		
55	5·10	13·41±0·2	13·6		
56	5·10	13·96±0·25	14·1		
57	5·19	12·97±0·1	12·8	12·7	0·05
58	5·19	12·75±0·1	12·7		
59	5·19	12·92±0·1	12·8		
60	4·48	0·75±0·3	0·57	0·4	0·1
61	4·48	0·47±0·1	0·31		
62	4·48	1·04±0·5	0·84		

λ = wavelength (mm.); $\bar{\alpha} \pm \epsilon$ = measured attenuation (db/km.); $\bar{\alpha}_c$ = corrected attenuation (db/km.); $\bar{\alpha}_m$ = mean $\bar{\alpha}_c$ for each wavelength; P.E. = probable error (±db/km.).

These are values measured under differing atmospheric conditions and have now to be reduced to correspond to a standard atmosphere. The first correction is for water vapour. In his analysis of absorption by water vapour Van Vleck (1947 b) treats the attenuation in two parts: (*a*) that due to the absorption line at 1·35 cm. wavelength, and (*b*) the combined residual effect of other lines of much shorter wavelength. The theoretical values of these for the range of interest are shown as curves (*a*) and (*b*) of figure 3 for air containing 1 gm. of water per cubic metre. From a discussion of the experimental results of several workers at wavelengths around 1·3 cm. Van Vleck concludes that the theoretical values for (*a*) are correct, but for (*b*) should be increased by a factor of about 4. The sum of the two parts, with this modification, is given in figure 3, curve (*c*), and these values are used for the reduction of the present data. An attenuation appropriate to the actual concentration G gm/m³ of water vapour in the air, determined from the humidity, is subtracted from each measured attenuation rate. The values of G vary between 5·1 and 16 gm/m³, corresponding to attenuation rates between 0·13 and 0·31 db/km.

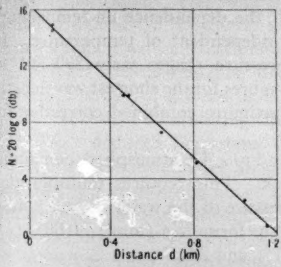

Figure 2. Typical set of observations plotted to give the attenuation constant as the gradient of a straight line.

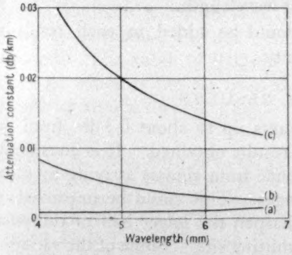

Figure 3. Attenuation due to water vapour, per gramme H_2O per m³, at 293° K., 760 mm. Hg.

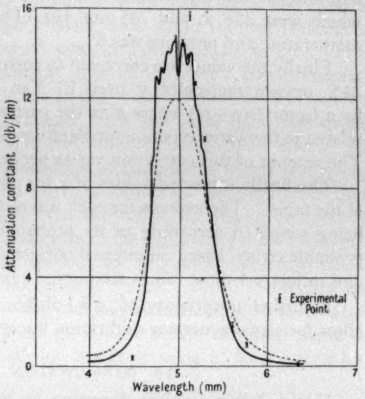

Figure 4. Measured values of attenuation due to oxygen, at 293° K., 760 mm. Hg.

Theoretical curves $\begin{cases} \text{———} \Delta\bar{\nu}=0\cdot02\ \text{cm}^{-1}. \\ \text{- - -} \Delta\bar{\nu}=0\cdot05\ \text{cm}^{-1}. \end{cases}$

The remainder must now be corrected for temperature and pressure, the standards used being 293° K. and 760 mm. Hg. The theoretical formula given by Van Vleck (1947 a) for the attenuation due to oxygen is of the form $\alpha = FN/T$, where N is the number of molecules of oxygen per unit volume, T the absolute temperature, and F a complex function of individual resonance frequencies. It is known (Beringer 1946) that the collision cross-section is substantially the same for O_2–O_2 and O_2–N_2 impacts. If we assume in addition that O_2–H_2O impacts are equally effective, then the attenuation will be proportional to the total number of molecules per unit volume and hence to the total pressure p. This is a reasonable assumption since the kinetic theory collision diameters for O_2–H_2O and O_2–N_2 impacts are respectively 4·1 A. and 3·7 A.; other work in this field has shown that collision diameters are little different from those calculated by kinetic theory. As N varies inversely with T, the dependence on temperature is approximately as T^{-2}, F being relatively independent of temperature. In these measurements the temperature and pressure ranges were 290–301° K. and 755–763 mm. Hg, with the exception of the figures for the shortest wavelength, which were 278° K. and 745 mm. Hg. The maximum combined correction for temperature and pressure was 8%.

Finally the values are corrected to correspond to a dry atmosphere containing 21% oxygen molecules as used by Van Vleck. This requires multiplication by a factor $p/(p-p')$ where p' is the partial pressure of the water vapour, and is related to the water vapour concentration G by the formula p' (mm.) $= 0{\cdot}0035\,GT$. The amount of this correction varies between 1% and 2%.

The finally corrected values of $\bar{\alpha}$ for each run are given in the fourth column of the table. The average for each wavelength group is then taken, each reading being weighted according to its probable error. The final results with their probable errors, based on internal consistency, are shown in the last two columns, and in figure 4 these values are plotted against wavelength.

A further uncertainty of ±0·1 db/km. should be added to each result to allow for signal generator calibration uncertainty.

§ 6. DISCUSSION OF RESULTS

Figure 2 shows points diverging by amounts up to about 0·3 db. from the linear relationship, and this is typical of the results obtained. It is considered that part of this variability is due to interference from signals arriving by other than the direct path, even on the shorter ranges. This could be improved by re-design of the receiving aerial system to sharpen the lower half of the polar diagram, or preferably by using a higher transmitting site. Some of the variation must also be attributed to slow changes in the transmitter output and to a certain amount of instability in the distorter and mixer crystals, which, requiring extremely light contact and having little whisker length to allow flexibility, cannot be so robust as the capsule crystals familiar at longer wavelengths.

As regards dependence on external conditions, temperature is the most important, and a change of 1° C., for example, would produce a change of about 0·1 db/km. for the larger values of attenuation. There will be variations in temperature over the path, characterized by horizontal and vertical gradients and irregular short-period variations. These have been discussed by Sheppard (1946), and from his paper it is clear that the difference between the temperature at the transmitter and receiver heights may easily vary by 1° C. during the day, and short-period variations at a fixed point up to ±1° C. are usual. These uncontrollable variations are probably the limiting factor in measurements of this type.

In figure 4 the solid curve is that calculated by Van Vleck for a line-breadth constant $\Delta\bar{\nu}$ of $0{\cdot}02\,\mathrm{cm}^{-1}$, and the values are obviously in good agreement with the curve. The spectroscopic determinations of the energy intervals from which the curve is calculated have not sufficient exactitude to allow details of its incipient fine structure to be plotted accurately, but the curve is a good approximation. That given here differs very slightly from Van Vleck's curve, in that spectroscopic values of higher accuracy by Babcock, quoted but not used by Van Vleck, have been used. The dotted curve in figure 4 is for $\Delta\bar{\nu}=0{\cdot}05\,\mathrm{cm}^{-1}$, a width too great to show any individual lines, and it is evident that the measurements do not support this value. The value of 15·7 obtained for α at 5·13 mm. wavelength, which is well above the curve, may indicate a line-breadth constant rather less than $0{\cdot}02\,\mathrm{cm}^{-1}$, with a consequent increase in the amount of structure appearing.

Beringer's measurements, made in the laboratory over a similar range of wavelengths, are in general agreement with the values given here, but show a greater spread. Considering Beringer's work and some indirect evidence, Van Vleck deduced that about $0{\cdot}02\,\mathrm{cm}^{-1}$ was the best estimate of $\Delta\bar{\nu}$. The present work substantiates this. Here, as also in Beringer's measurements, it would appear that the curve should be rather wider on its long-wavelength side than the existing theory would indicate. On the other hand the single point on the skirt at the short-wavelength end indicates the opposite effect. In his paper Van Vleck remarks that "it should not be regarded as highly surprising if the detailed shape of the absorption curve is somewhat different from that calculated by means of our formulas". On the whole the points in figure 4 suggest that the curve should be moved slightly to the right along the wavelength axis, but there would appear to be no warrant for this on a theoretical basis. For the moment it can be said that the present results confirm the theory by a very direct method and add weight to the previous estimate of the line-breadth constant. The value of $0{\cdot}02\,\mathrm{cm}^{-1}$ for $\Delta\bar{\nu}$ gives an effective collision diameter of 2·8 A., which is not far from the collision diameter value of 3·6 A. given for oxygen by kinetic theory.

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Note on Nuclear Magnetic Resonance

By E. E. SCHNEIDER

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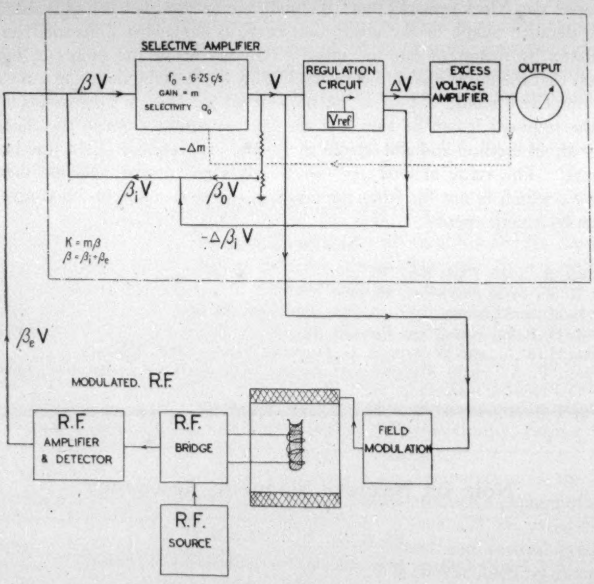
MS. received 6 *August* 1948; *read at Oxford Conference on Microwave Spectroscopy* 23 *July* 1948

I should like to report briefly on a general method of physical measurement which seems ideally suited for the detection of very weak nuclear magnetic resonances, and which promises to be very useful for the absolute measurement of relaxation times.

The main idea of the method is to increase the discrimination against noise by utilizing the extreme frequency selectivity of a continuously oscillating system maintained by non-linear regeneration. According to the classical work of Appleton and van der Pol the inherent damping of the original resonator must be overcompensated by the regenerative action so that the effective damping λ in the maintained state becomes negative and the reproduction factor k, giving the ratio of the regenerated to the original oscillating quantity, becomes greater than unity. For given amplitude the values of $k-1$ and λ decrease with decreasing non-linearity. The selectivity of the system with respect to driving forces of different frequencies around the oscillation frequency is essentially given by $|\lambda|$ and can, therefore, be very large if the non-linearity is small.

Such a non-linear system can serve for the measurement of a physical effect if the effect itself can be made to provide an additional regenerative action. This external regeneration will add up to the internal regeneration already present and so change the oscillation amplitude. More convenient is the use of a regulation method to measure the external effect by the compensating change of the internal regeneration required to keep the amplitude constant.

The diagram shows the application of this method to the study of nuclear magnetic resonance. The oscillating system consists of a selective very low frequency amplifier with positive feedback providing the main internal regeneration. The oscillation amplitude is regulated by a negative feedback contribution resulting from a deviation of the output from a reference value. This deviation or excess voltage indicates the magnitude of the effect under observation since it is a measure of the change in β_i compensating the external feedback. The



SELECTIVE AMPLIFIER
f₀ = 6·25 c/s
GAIN = m
SELECTIVITY Q₀
βV
V
REGULATION CIRCUIT
Vref
ΔV
EXCESS VOLTAGE AMPLIFIER
OUTPUT
-Δm
βᵢV
β₀V
-Δβᵢ V
K = mβ
β = βᵢ + βₑ
βₑV
MODULATED. R.F.
R.F. AMPLIFIER & DETECTOR
R.F. BRIDGE
R.F. SOURCE
FIELD MODULATION

oscillator output is used to modulate the magnetic field of the otherwise conventional nuclear resonance set-up. With the mean magnetic field adjusted near resonance the radio frequency is, therefore, modulated at the low oscillator frequency and the detected output from the R.F. receiver forms a feedback voltage which combines vectorially with the internal feedback voltage. Its phase depends on the position on the magnetic resonance curve of the mean magnetic field, on the modulation index, and on the relaxation time and the R.F. intensity.

With a system of this type values of $k-1$ of the order of 10^{-3} have been obtained and changes of k of less than 10^{-3} have been detected. Since $\Delta k=\Delta\beta m$, $V_i=\Delta\beta V=\Delta kV/m$, and the gain $m=5\times10^4$ and the output voltage $V=10$ v., this means that voltage changes at the low-frequency oscillator input of smaller than 2×10^{-7} v. are observable across an input resistance greater than 10^8 ohm. When using the R.F. arrangement, providing an additional gain of 10^3, observable deflections of the final output meter have been obtained by a modulation component of less than 10^{-9} v. across the R.F. input resistance of 3×10^4 ohm, i.e. by an available input power of less than 10^{-23} watt. This high sensitivity in the presence of thermal noise could be interpreted in terms of a bandwidth of less than 1/400 c/s. It is very doubtful, however, whether the concept of a bandwidth for noise has any sense in a non-linear system. It is certainly not correct to consider the reciprocal of the bandwidth as the response time which can here not be defined as the time constant of a simple exponential decay or build up of the response. The time actually taken by the system to respond to the small input voltage changes just quoted is of the order of several seconds and not minutes as would follow from the bandwidth.

The possibility of obtaining very high sensitivities, very high "signal to noise ratios", without a sacrifice in the speed of observation is the most important achievement of the new method and a direct consequence of its essential non-linearity.

CORRIGENDA

"A Radio-sonde Method for Atmospheric Potential Gradient Measurements", by R. E. Belin (*Proc. Phys. Soc.*, 1948, **60**, 381).

I wish to point out that the work as described under the above title was carried out at the Auckland University College, New Zealand (and not at the address stated in the paper) under the supervision of Professor P. W. Burbidge and Dr. K. Kreielsheimer.

The latter has kindly pointed out an error in equation (3), p. 385, which should read $i=i_0(P_0/P)^{1\cdot6}$, and a better explanation of figure 6, viz., the figure suggests a negative distribution of charge residing both at the top and the bottom, with a positive distribution near the centre of the cumulo-nimbus cloud, while the fracto-stratus formation may have a positive charge at the bottom and negative at the top.

It should also have been emphasized that the formulae used in the paper are only approximate.

"Travelling Wave Linear Accelerators", by R. B. R.-Shersby-Harvie (*Proc. Phys. Soc.* 1948, **61**, 255).

In figure 9 the ordinate scale is too great by a factor of $\sqrt{3}$.

ABSTRACTS OF PAPERS READ AT OXFORD CONFERENCE

23rd and 24th July 1948

Spectroscopy at Radio and Radar Frequencies

By C. J. GORTER
Leiden

Brief opening remarks (*see this issue*, p. 541),

The Theory of Line Breadths in Microwave and Radio Frequency Spectra

By J. H. Van VLECK
Harvard

Three mechanisms for line broadening may be distinguished : (*a*) collision broadening of the Lorentz type, applicable to the microwave spectra of gases, (*b*) collision modulation of intramolecular nuclear spin–spin coupling, whose theory has been developed by Bloembergen, Pound and Purcell, and (*c*) the adiabatic broadening by dipolar interaction of the magnetic resonance of regular crystals, devoid of molecular rotation. The present paper is concerned primarily with (*c*), the simplest process of the three. The eigenvalue problem is still too complicated to permit calculation of the detailed line profile, but the computed mean square and mean fourth power frequency deviations agree excellently with Pake's measurements on the magnetic resonance of the F nucleus in CaF_2, both as regards absolute magnitude and directional dependence. The line contour in CaF_2 is almost gaussian. On the other hand, the observed half-widths of microwave absorption lines arising from electronic spin in paramagnetic salts are considerably smaller than calculated from the theoretical mean square if a Gauss distribution is assumed. The discrepancy is shown to be caused by the phenomenon of " exchange narrowing ", wherein the fluctuations produced by exchange forces spoil the coherence and efficiency of the dipolar coupling. (*To be published in Physical Review*, 1948.)

Experimental Investigations of Paramagnetic Resonance Lines

By J. H. E. GRIFFITHS
Oxford

Paramagnetic resonance in $CuSO_4.5H_2O$ has been investigated at wavelengths of 3·04 cm., 1·23 cm. and 0·85 cm. Two features of considerable interest are observed, first, that under certain conditions, the absorption line is considerably narrower than would be expected from the normal magnetic interactions of the ions, and second, that at the longer wavelengths only one line is observed whereas, at certain orientations of the crystal in the external magnetic field, two lines should be observed. The two lines are clearly resolved at 0·85 cm. wavelength. A qualitative explanation of these results is given in terms of exchange interaction. These results are also compared with those for $CuSO_4.K_2SO_4.6H_2O$ in which the exchange interaction appears to be considerably weaker. (*See* D. M. S. Bagguley and J. H. E. Griffiths, *Nature, Lond.*, 1948, **162,** 538.)

Theory of the Line Breadth in Copper Sulphate

By M. H. L. PRYCE
Oxford

The observations on copper sulphate ($CuSO_4 . 5H_2O$) suggest that the rapid exchange between neighbouring ions so reduces the effect of the internal magnetic field that the breadth due to magnetic interaction of the ions is only a minor part of the observed breadth. The main breadth can be explained as arising from exchange between crystallographically dissimilar ions.

In a crystal in which all the paramagnetic ions are similar, exchange cannot, in the absence of magnetic interaction, have any broadening effect on the resonance. In copper sulphate, however, there are two kinds of ions, differing in the orientation of the atoms in their neighbourhood. Exchange between the two leads to a broadening. A quantitative calculation of the effect is possible for low frequencies (up to about 10 000 Mc/s.). It shows that for certain orientations of the magnetic field the breadth increases as the square of the frequency. The dependence on orientation of the field is given by a formula containing three undetermined parameters. (*See Nature, Lond.*, 1948, **162,** 539.)

Paramagnetic Resonance at Low Temperatures

By B. BLEANEY
Oxford

A general survey has been made of salts of the iron group by the method of paramagnetic resonance to establish the conditions under which absorption lines are detectable. At room temperature and 3 cm. wavelength lines are observed in salts of $3d^3\,Cr^{+++}$, $3d^4\,Cr^{++}$, $3d^5\,Fe^{+++}$ and Mn^{++}, and $3d^9\,Cu^{++}$. In $3d^1\,Ti^{+++}$, $3d^6\,Fe^{++}$, $3d^7\,Co^{++}$ the spin–lattice relaxation time is too short at room temperature but resonance can be observed at sufficiently low temperatures. Short wavelengths are required for $3d^8\,Ni^{++}$ (and also Fe^{++}) because of the large separation of the spin levels in zero magnetic field. These effects are associated with the splitting of the orbital levels caused by the electric field of the crystalline lattice.

Single crystals of four chrome alums have been studied. At room temperature the spectra can be interpreted assuming the lattice produces a small trigonal electric field superimposed on a cubic field. The spin levels consist then of two doublets with a separation varying from 0·12 cm^{-1} for the potassium alum to $0{\cdot}16_5$ cm^{-1} for the rubidium alum. As the temperature is reduced the splittings decrease slightly in rubidium and caesium alums while in potassium and ammonium alums a considerable initial decrease is followed by the appearance of different types of spectra at liquid oxygen temperatures. Each of these two alums then exhibits two splittings, presumably associated with different ions in unit cell. (*See this issue*, pp. 542, 551.)

Pressure Broadening of the Ammonia Spectrum

By B. BLEANEY
Oxford

At centimetre wavelengths the dominant contribution to the width of gaseous spectral lines is due to collision broadening. It is thus possible to study collision broadening over a wide range of pressures, and the absorption due to the strong inversion spectrum of ammonia near 1 cm. wavelength has been measured over the range 6 atmospheres down to a few hundredths of a millimetre. The main features are :

(*a*) From measurements of the line widths at 0·5 mm. Hg, the collision diameter is found to vary from 9 A. to 14 A., which is considerably greater than the kinetic theory diameter, 4 A. This, and the regular change in width with the rotational state of the molecule are explained in terms of dipole–dipole interaction. The cross-sections for collisions with non-polar gases are approximately equal to the kinetic theory values.

(*b*) At pressures up to 10 cm. Hg, the absorption curve agrees closely with that calculated on the assumption that the line width varies linearly with the pressure, but at higher pressures the centre of the absorption shifts to lower frequencies and the line width increases less rapidly than the pressure. Above two atmospheres the inversion frequency becomes zero and the collision diameter tends to a new constant value of 8 A.

(*c*) At pressures below a few tenths of a millimetre a " saturation " effect is observed, since the collision frequency becomes too low to counteract the disturbance of thermal equilibrium produced by the radiation. Comparison with theory shows that the thermal relaxation time is equal to the mean time between collisions. (*See* Bleaney, B., and Penrose, R. P., *Proc. Phys. Soc.*, 1948, **60,** 83.)

Microwave Absorption in Gases

By M. H. L. PRYCE
Oxford

In microwave spectroscopy one can use coherent incident radiation which is monochromatic to a high degree, and the breadth of absorption lines in gases is due either to molecular collisions or to the intensity of the incident radiation, Doppler effect and natural line breadth being negligible. Lorentz's classical theory of pressure broadening needs refining in two interesting respects. For very broad lines Lorentz's hypothesis that collisions leave the molecular oscillators with random phase needs to be modified. This has been done by Van Vleck and Weisskopf, and by Fröhlich, but the experimental material suggests that a further refinement is required, in which the collision process is considered in greater detail.

For high intensities of the incident radiation the Boltzmann distribution of the energy levels between which the microwave transitions are taking place is disturbed, giving rise to a broadening. This is a phenomenon not found in optical spectra. It is essentially a quantum phenomenon, and can be calculated by a simple extension of the usual absorption calculations. This theory will be discussed. (*To be published in Proc. Phys. Soc.*)

Atmospheric Absorption of Millimetre Waves

By H. R. L. LAMONT
General Electric Co.

The paper describes measurements of the attenuation, due to molecular absorption in oxygen, of electromagnetic radiation in the region of 5 mm. wavelength. The attenuation was measured as it occurs in the atmosphere, using paths up to 2 km. long over flat ground, with a fixed elevated transmitter and a mobile superheterodyne receiver. The relative signal strength at any point was determined by direct comparison with a 60 Mc/sec. signal generator having an accurately calibrated attenuator. Ten wavelengths between 6·34 and 4·48 mm. were used, making a tolal of thirty-five runs, each comprising about seven readings.

The mean attenuation derived from each set of readings was corrected for water vapour attenuation, and then reduced to a value appropriate to an atmosphere at 20° c. and 760 mm. Hg. The final attenuation values range up to a maximum of 15·7 db/km., with probable errors between 0·1 and 0·2 db/km. They are in good agreement with Van Vleck's theoretical curve for a line-breadth constant $\Delta v/c$ of 0·02 cm^{-1}, in confirmation of estimates of the line-breadth derived from other methods. (*See this issue,* p. 562.)

Nuclear Effects in Microwave Absorption in Molecules

By A. G. HILL
Massachusetts Institute of Technology

Some twenty-five lines of the microwave spectrum of the linear molecule OCSe have been observed by Strandberg and others at M.I.T. Lines associated with the six stable isotopes of selenium and with C^{12}, C^{13} and O^{16} have been measured in the ground vibrational state. In addition some of the stronger lines have been observed for the symmetric vibrational and bending states. Most of the observations have been made for the $J=2\rightarrow3$ transition, lying in the region near 25 000 Mc/s., but in addition a few lines have been observed for the $J=5\rightarrow6$ and $J=6\rightarrow7$ transitions. There is no evidence of quadrupole coupling for the Se^{77} isotope greater than 0·5 Mc/s., and from a standpoint of reasonableness one concludes that the spin of Se^{77} is $\frac{1}{2}$. Mass differences for the Se isotopes have been obtained assuming the values of Se^{74} and Se^{82}. Stark effect measurements enable one to calculate the dipole moment. Measurements of relative intensities for the ground state and for the bending state enable one to predict a vibrational line at 474 cm^{-1}. Lord and McDonald have subsequently detected a transition at 464 cm^{-1}. Relative intensity measurements are in very good agreement with known data on relative isotopic abundance. (*To be published in Physical Review or Proc. Phys. Soc.*)

Magnetic Relaxation of Nuclei

By N. BLOEMBERGEN
Leiden

Experiments on nuclear magnetic resonance in crystals of CaF_2 and $KAl(SO_4)_2.12H_2O$ show that relaxation times are at least 10^5 times shorter than predicted by Waller's theory, and are reduced strongly by small amounts of paramagnetic impurities. The dependence of the relaxation time on temperature has been investigated between 300° K. and 1° K. At the lowest temperature relaxation times as long as half an hour have been observed.

A new theory of the heat contact between nuclear spins and lattice vibrations has been developed. The energy absorbed by the spins is transported toward the impurities by a diffusion process. The impurities act as sinks, through which the energy is transferred to the lattice. If the diffusion is the limiting factor in the energy transfer, the relaxation time is independent of temperature, and it is a decreasing function, if the impurities are the bottleneck. The detailed behaviour will be very sensitive to the nature and amount of the paramagnetic impurity. The theoretical results agree, at least in order of magnitude, with the experimental data.

Experiments on Nuclear Resonance at Low Temperature

By B. V. ROLLIN
Oxford

Further measurements on nuclear paramagnetism at low temperatures have been made using the magnetic resonance absorption method. Experiments have been made on single crystals of calcium fluoride at temperatures between 20° K. and 0·2° K. in order to determine the effect of temperature, magnetic field strength and paramagnetic impurities on the spin–lattice equilibrium time of the fluorine nuclei.

Nuclear resonance has been observed in copper and aluminium at 1·1° K.

Measurements have been made on solid hydrogen at temperatures down to 1·1° K. A splitting of the resonance is observed at the lowest temperatures. (*To be published in Rep. Prog. Phys.*, Vol. XII, 1949.)

Effects of Nuclear Electric Quadrupole Moments in the Radio-frequency Spectra of Solids and Liquids

By R. V. POUND
Harvard

Several cases have been reported of nuclear magnetic resonance absorption in liquids, in which a fluctuating interaction between the nuclear electric quadrupole moment and the gradient of the electric field at the nuclei shortens the relaxation time and broadens the resonance line. Such an effect is very pronounced in the resonances of bromine, iodine and gallium in solutions. In crystalline solids of lower than cubic symmetry the absorption line should have a structure with a multiplicity depending on the spin of the nucleus. Evidence of such an effect was found in the Li^7 resonance in powdered Li_2CO_3 and $Li_2SO_4 . H_2O$, and a definite triplet line was observed in a properly oriented single crystal of $Li_2SO_4 . H_2O$. In a corundum crystal, the line from Al^{27} was found to have the five components characteristic of a spin of 5/2 and the splitting was not very small compared to the centre frequency used. Accordingly, the first order terms in the perturbation energy were not sufficient fully to describe the interaction. Excellent agreement was found between calculations of the interaction carried to the third order and the experimental observations of the dependence of the line frequencies on the orientation of the crystal in the magnetic field. (*To be published in Proc. Phys. Soc.*)

Measurements of Relaxation Times

By L. E. DRAIN
Oxford

This direct method of determining nuclear spin–lattice relaxation times which are comparable with the period of the modulating field is based on Bloch's nuclear induction experiment. As the magnetic field is varied through resonance, the sizes of the signals on the forward and backward sweeps, depend on how long is spent above and below resonance and upon the relative magnitudes of the relaxation time and the period of sweep.

Under certain conditions, a simple relation holds between the size of the signal, its position on the trace and the relaxation time. A quick and convenient procedure is to observe the point at which one signal disappears. This gives the relaxation time directly. (*To be published in Proc. Phys. Soc.*)

Investigations of Ferromagnetism in the Microwave Region

By J. H. E. GRIFFITHS
Oxford

Measurements of ferromagnetic resonance at 1·23 cm. wavelength in thin films of nickel are described. The films were obtained by evaporating nickel on to mica. The results may be expressed in terms of the equation $g\beta(BH)^{\frac{1}{2}} = h\nu$ obtained by Kittel, where β is the Bohr magneton, B and H the magnetic induction and field strength, ν the frequency and $g=2$ if the resonance is due to electron spin only. Previously reported measurements on bulk material have given values for g of about 2·15. Thin nickel films ($<0{\cdot}5\mu$ thick) give $g=2{\cdot}00 \pm 0{\cdot}02$ while films greater than $0{\cdot}8\mu$ thick give $g=2{\cdot}14 \pm 0{\cdot}02$ in agreement with the values for the bulk material. Intermediate values are obtained in the transition region. (*To be published in Nature, Lond.*)

Microwave Resonance Absorption in Ferromagnetic Materials

By C. KITTEL
Bell Telephone Laboratories

The ferromagnetic resonance absorption effect was discovered by Griffiths, who observed that the energy loss in a microwave cavity containing ferromagnetic material shows a resonance type behaviour as a function of the static magnetic field intensity applied to the cavity. The effect basically is the result of electron spin resonance absorption, and it has been shown by the present author that the resonance relation between frequency and applied field is $\omega=g(e/mc)(BH)^{\frac{1}{2}}$ for the particular case of a plane specimen with the applied field parallel to the plane. The relation may be derived classically, or else, on quantum mechanics using the Hamiltonian $H=-\vec{M}\,.\,\vec{H}+2\pi M^2 y$. The resonance relation depends, according to theory, on the specimen shape, the field orientation, and, in the case of a single crystal, on the orientation of the crystal axes.

These predictions of the theory are confirmed in several cases by the measurements of Yager, Kip and Arnold, and Hewitt. In several materials it appears that the *g*-value is in the neighbourhood of 2·2 : this conclusion is particularly well confirmed in the case of Supermalloy. There is a special interest in Heusler alloy, but preliminary measurements by Yager do not give a conclusive *g*-value because of the width of the resonance. (*See Physical Review*, 1948, **73**, 155.)

Microwave Absorption in Metals at Low Temperatures

By J. C. SLATER
Massachusetts Institute of Technology

Measurements of the surface resistance and reactance of lead, tin and copper, at a wavelength of 1·25 cm., and temperatures down to about 2° K., have been carried out by a group at M.I.T. using a resonant cavity method, and the programme contemplates measurements on a number of other metals. The results are qualitatively similar to Pippard's measurements at longer wavelength. The decrease in bulk resistivity in going to very low temperatures is not accompanied by a corresponding decrease in surface resistance, as would be calculated by the classical theory of skin effect. Furthermore, below the superconducting transition, the surface resistance does not go to zero, but decreases at first rapidly, then slowly, remaining finite at 2° K. These results are compared with the theories of London, Pippard, and Sondheimer and Reuter, based on the assumption that the electronic mean free path becomes greater than the classical skin depth at very low temperature and high frequency. Comparison with Pippard's experiments at longer wavelengths gives fairly good, but not complete, checks of the theory, both for normal and superconductors.

Microwave Experiments with Superconductors

By A. B. PIPPARD
Royal Society Mond Laboratory

The surface resistance of normal and superconducting tin has been measured at a wavelength of 3·2 cm. Comparison with earlier measurements at 25 cm. shows that just above the superconducting transition temperature the resistance varies as $\omega^{2/3}$, in accordance with the theory of Reuter and Sondheimer. From dimensional arguments, based on the " two-fluid " concept of superconductivity, as exemplified by the theories of F. and H. London, Heisenberg, and Born and Cheng, relations are deduced for the expected behaviour, as the frequency is changed, of the superconducting resistance just below the transition temperature and at much lower temperatures. These relations are not confirmed by the experiments, and up to now no additional resistive mechanism has been found to explain the discrepancies. It is possible, therefore, that the phenomenon of superconductivity may not be explicable in terms of a two-fluid model. (*Not to be published.*)

Vapour Pressure of Helium 3

BY J. DE BOER
Amsterdam

The differences in the properties of isotopic substances like He^3 and He^4 and like H_2, DH, D_2 and DT are due to the strong influence of quantum effects on the properties of these light substances.

An exact theoretical treatment of these quantum effects meets great difficulties, resulting from the impossibility of treating the many particle problem ; in the solid state the large amplitude of the zero-point vibration makes application of the usual theory of solids very doubtful, and in the liquid or condensed gas state, the known difficulties of treating fluid systems are added to the quantum effects.

The magnitude of these quantum effects in the condensed noble gases and the hydrogens can be determined in a semi-empirical way, avoiding the difficulties mentioned above by application of the quantum theory of the law of corresponding states. All physical quantities are reduced by means of " molecular units ", which are defined in terms of the length σ and the energy ϵ, characterizing the intermolecular potential field, and the values of which are known for the sequence of substances : Xe, Kr,A, Ne, H_2 and He.

In classical theory the reduced equation of state should be the same for all these substances. Quantum mechanical considerations show, however, that the reduced equation of state should be a function of the " quantum parameter " $\Lambda^* = h/\sigma\sqrt{(m\epsilon)}$ in which the mass m of the molecules occurs. Indeed, the reduced values of all physical quantities characteristic of condensed substances : molar volume and heat of vapourization, triple point—and critical temperature, Debye-temperature etc., appear to be smooth functions of this quantum parameter, the limiting value for $\Lambda^* \to 0$ being the classical value.

This dependence of reduced physical quantities on Λ^* can be used to calculate unknown values of these quantities for isotopic substances He^3 or HD, D_2 and DT, from the known experimental values of these quantities for the main isotope He^4 or H_2 respectively, because of the fact that molecules of isotopic substances have the same interaction. In this way the properties of liquid He^3 have been predicted, although the fact that in this case the functional dependence on Λ^* has to be extrapolated might lead to appreciable errors. The results are discussed in detail.

The critical temperature and pressure should be 3·5° to 3·1 °K. and 1·3 to 0·95 atm. At 2° K. the vapour pressure of He^3 proves to be more than six times larger than that of He^4. The vapour pressure formula is given and compared with that of He^4. The results of the measurements of Fairbank *et al.* are discussed and prove to agree with the expected theoretical values. The deviations from Raoult's law will be discussed in detail. (*To be published in Physica.*)

The Separation of Helium 3 by Liquid Phase Methods

BY J. G. DAUNT
Ohio

A general review is given of the experimental work carried out on the properties of the helium isotopes in the liquid phase. Detail is given of the experiments which established that the superfluid flow of matter in liquid helium II is one which does not involve the rare isotope He^3 and that in consequence a new method of isotopic separation is available.

An account is given of experiments which established the existence of an osmosis occurring between solutions of He^3 in He^4 of different concentrations and which have been carried out on the measurement of the distribution coefficient in such solutions.

An account is also given of the technique of separation now set up for continuous operation.

Experiments on Helium Films

By J. B. BROWN
Oxford

Experiments have been made to investigate the dependence of Helium film transport rate on the vapour pressure over the film. The film has been isolated from the bulk liquid and the vapour over it by a technique employing capillary leaks in wire packed tubes.

A study has also been made of anomalously high film transport rates through holes in surfaces. The effect has been related to previously observed high transport rates near the rim of cylindrical beakers and it is seen to have a practical bearing on the design of baffles used for attaining low temperatures by pumping the vapour over liquid helium.

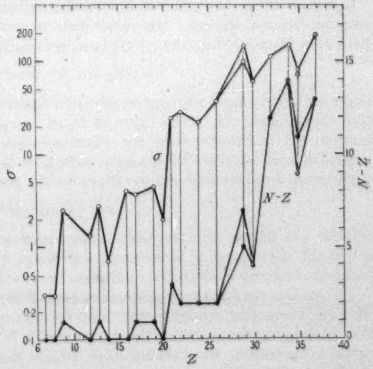
LETTERS TO THE EDITOR

Nuclear Photo-Effect and Isotopic Number

The first measurements of the yields of neutron nuclear photo-effects (γ, n) by Bothe and Gentner have been refined and extended to a greater number of elements by Scherrer and his collaborators. Recently, Wäffler and Hirzel (1948) have published new values of the relative (γ, n) cross-section σ of 35 isotopes with atomic numbers Z reaching from 7 to 79 for the lithium γ-rays. In the figure these values of σ are represented up to $Z=37$ (upper line, logarithmic scale).

The general increase of σ with increasing Z may be explained by the fact that the level density of nuclei for the high excitation energy in question increases rapidly with Z. In order to account for the remarkable rise of σ for $Z>20$, Wäffler and Hirzel tentatively assume a corresponding rise of the level density in this region of Z.

Looking for further reasons for this sudden rise and for the fluctuations of σ for the light nuclei, I have compared σ with the isotopic number $N-Z$ of those isotopes from which the (γ, n) reaction starts (N=neutron number). The lower line in the figure represents $N-Z$ on a linear scale. A striking parallelism is seen which is only interrupted in the two cases of ^{32}S and ^{46}Ti, so that it can hardly be regarded as accidental. Beyond $Z=40$ the correlation between the two quantities is no longer evident.



Relative (γ, n) cross-section σ, according to Wäffler and Hirzel, and isotopic number $N-Z$, plotted against the atomic number Z.

One might first attempt to explain this correlation by means of energy considerations, since light nuclei with $N-Z \geqslant 1$ have a comparatively low neutron binding energy and therefore the neutron emission probability of these nuclei for the excitation energy 17 MEV. is comparatively large. It should then be assumed that nuclei with a smaller isotopic number would emit a correspondingly greater number of protons and α-particles. Too little is known about the ratios of the emission probabilities of neutrons, protons and α-particles for the single isotopes of the light elements up to Ca, so that this assumption cannot as yet be checked directly. However, according to the measurements of the (γ, n)

and (γ, p) cross-sections by Wäffler and Hirzel (1947, 1948), it is known that for nuclei heavier than Ca the (γ, n) reaction is always by far the most probable one, independently of the isotopic number. Moreover, it is doubtful whether the (γ, p) process may be regarded as competing, according to Bohr's conception, with the (γ, n) process (Hirzel and Wäffler 1947, Schiff 1948, Jensen 1948).

Thus the simple consideration of binding energies is not sufficient to account for the correlation between σ and $N-Z$. One may then assume the nuclear structure to be also responsible. Accordingly, light nuclei should have a structure which makes them, other things being equal, more capable of absorbing the Li γ-rays and (or) emitting neutrons the greater their " neutron excess " $N-Z$. Here the domain of " light " nuclei should, according to the figure, be extended to about $Z=40$.

The author is indebted to Professor Gentner for valuable discussions.

Physical Institute of the University
of Freiburg, Germany.
September 1948.

P. Jensen.

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Schiff, L. I., 1948, *Phys. Rev.*, **73**, 1311.
Wäffler, H., and Hirzel, O., 1948 *Helv. phys. Acta*, **21**, 200.

Validity of Two-Dimensional Design of Synchrotron Pole-Faces

In a recent note Davy (1948) has considered the design of a synchrotron pole-face using a two-dimensional theory. His result may be generalized by the statement that, to obtain a field $H_z=H_0(x/r_0)^{-n}$ on the $z=0$ plane, pole-faces are required which satisfy the equation

$$r=\text{constant}\times\operatorname{cosec}^{1/1-n}[(1-n)\theta]. \qquad \ldots\ldots (1)$$

x and z are cartesian coordinates as shown in figure 1 (b), while r and θ are the corresponding polar coordinates; H_0 is the value of H_z at $x=r_0$, r_0 being the equilibrium orbit radius. Equation (1) is simpler than the exact solution of the three-dimensional problem by Coggeshall and Muskat (1944), and, indeed, it is permissible to approximate still further by assuming θ to be small and to obtain for the pole-faces the equations

$$z=\pm\text{constant}\times x^{n}. \qquad \ldots\ldots (2)$$

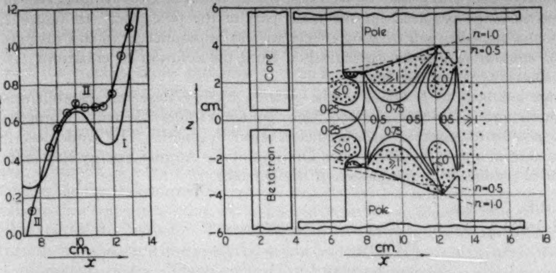
Equation (2) differs imperceptibly from equation (1) in practical designs and has been used as the starting point in the empirical design of pole-faces of the 30, 140 and 375 Mev. synchrotrons being constructed in Britain (Fry *et al.* 1948).

To examine the validity of a two-dimensional design, the exact profile could be calculated, following Coggeshall and Muskat, and the errors of the simpler design assessed. The effect of fringing fields and of the exciting coils is important, however, and in a betatron-started synchrotron, the betatron-core affects the field between the main poles. It is therefore more realistic to compare two-dimensional measurements in an electrolytic tank (Peierls 1946) with measurements on a three-dimensional magnet. As an example, figure 1 (a), curve I, shows measurements of n, taken along $z=0$ in a two-dimensional electrolytic tank, on the pole-faces of the 30 Mev. synchrotron. These measurements should be compared with the magnetic measurements shown by curve II. The differences between the two- and three-dimensional measurements are not very marked, and a sufficiently correct design can be made from two-dimensional electrolytic tank measurements. It is not possible, in any case, to obtain a constant value of n over the entire area between the pole-faces, as is shown by the " iso-n " lines of figure 1 (b) ; the important consideration is the region where stable orbits are possible, i.e. the regions for which $0\leqslant n\leqslant 1$. It should be noted that the values of n in the figures are defined by

$$n=-(x/H_z)(\partial H_z/\partial x). \qquad \ldots\ldots (3)$$

This is the most convenient definition in experimental work, although it differs from that used in equations (1) and (2) unless x/r_0 is near unity and θ is small.

The 30 MeV. synchrotron pole-face gives perhaps the greatest divergence between two- and three-dimensional measurements which is likely to be met in practice; the pole-face extends over a radial distance of more than half the equilibrium orbit radius and θ takes values up to 20°. By comparison with the 30 MeV. synchrotron the higher energy machines give a very small discrepancy between two- and three-dimensional measurements. Thus the 375 MeV. synchrotron pole-face shown in figure 2, together with the " iso-n " contours,



(*a*) (*b*)

Figure 1. Comparison of electrolytic tank and magnetic measurements for 30 MeV. synchrotron.

(*a*) Curve I shows two-dimensional measurements in electrolytic tank.
Curve II shows three-dimensional magnetic measurements, dots indicating D.C. measurements and crosses A.C. measurements.

(*b*) Pole-face profile showing lines of constant n from electrolytic tank measurements. The broken lines show the pole-face contours given by equation (1). The dotted regions are those where the electron orbits are unstable.

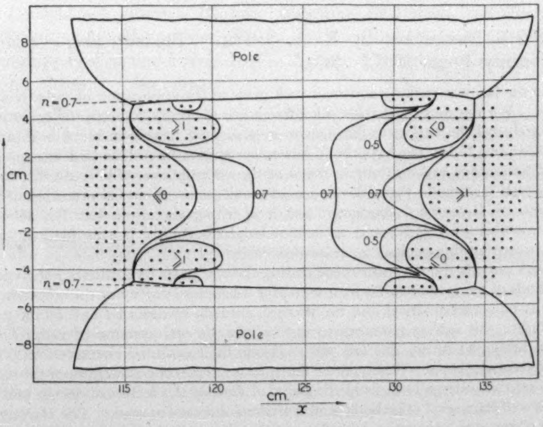


Figure 2. Electrolytic tank measurements for 375 MeV. synchrotron.

was designed by a two-dimensional electrolytic tank, but it has not been possible to measure any deviations between magnetic measurements and the measurements of figure 2.

Figures 1 (*b*) and 2 also show the comparison of the solutions of equations (1) or (2) with the empirical designs. In figure 1 (*b*) the solutions for $n=0{\cdot}5$ and $n=1{\cdot}0$ are given by the dotted lines, and it may be seen that the lip correcting for fringing has an important effect on the slope required for the main part of the pole-face. In figure 2, however, θ is very much smaller and fringing effects are less pronounced; in this instance, the main pole-face contour cannot be distinguished from the solution of equations (1) or (2) for $n=0{\cdot}7$, shown by the dotted lines.

We are indebted to J. Dain, of the Atomic Energy Research Establishment, and to J. F. Fowler and J. F. Smee, of the Metropolitan-Vickers Electrical Company, for permission to refer to magnetic measurements taken by them. The work described in this note was carried out in the Electronics Division of the Atomic Energy Research Establishment and is published by permission of the Director.

A.E.R.E. Division,
Telecommunications Research Establishment,
Malvern.
F. K. GOWARD.
J. J. WILKINS.

9th August 1948.

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REVIEWS OF BOOKS

Radio Aids to Navigation, by R. A. SMITH. Pp. xii+114. (Cambridge: University Press, 1947.) 9s.

During the last war striking advances took place in the application of radio techniques to navigation. For the most part these advances arose naturally through the growth of radar, and they included the various radar beacon systems, e.g. precision blind-bombing devices such as Oboe and G.H., the hyperbolic lattice systems such as Gee and Loran, and, later, the use of microwave radar to obtain maps of the country over which aircraft were flying. Much research leading to these developments took place in the laboratories of the Telecommunications Research Laboratory, and it is fitting therefore that Dr. R. A. Smith, himself a member of T.R.E., and an acknowledged authority in this field of knowledge, should have written this book.

The book records the advances made during the war years in both radar and continuous-wave methods of navigation and attempts to give a balanced survey of the whole field. The treatment is non-mathematical and no attempt is made to describe any of the systems in detail. Rather the author attempts to outline the various systems of radio (and radar) navigation developed during the last war, indicate their essential characteristics, and make critical inter-comparisons. Throughout, the author stresses the principles of the subject and comments on the systems both from the point of view of the technical design and from the point of view of the user. The book is well written and easy to read. The chapters include discussion of pre-war navigational systems, of the short and long range navigational aids—both radar and c.w.—developed during the last war, of microwave systems, of precision bombing aids, of radio altimeters, and of aids to instrument landing.

The book will be of interest to a wide range of readers, particularly all those who wish to learn something of the wartime developments in this field with a view to taking an intelligent interest in the applications in the field of civil flying and marine navigation. The book can be confidently recommended.

DENIS TAYLOR.

The Atom and its Energy, by E. N. da C. Andrade. Pp. viii+196. (London: G. Bell and Sons Ltd., 1947.) 10*s*.

The purpose of this book is to give an account, for the reader not trained in the physical sciences, of those principles and discoveries in physics which have made it practicable to derive energy on a useful scale from the nucleus. The book, the first few chapters of which embody much of an earlier book by the same author, begins with the considerations that led to the atomic theory of matter. It then reviews the evidence on which Rutherford based his hypothesis of the nuclear atom and that on which modern ideas of atomic and nuclear structure depend. The last chapters are devoted to nuclear transmutation, fission, and the chain reaction and its consequences. There is a clear exposition of the principles and advantages of the proposals made by the Lilienthal Committee for the international control of atomic energy.

Professor Andrade is wholly successful in achieving the aim that he sets himself. His book is remarkable for its comprehensiveness and clarity, a clarity that owes much to the aptness of the author's down-to-earth analogies and illustrations, based as they are on notions readily conceived by the layman. The reader's feet are kept firmly on the ground by the author's custom of translating into everyday terms quantities that are often left in theoretical, or at least unfamiliar, units. These qualities of Professor Andrade's, and his robust yet scholarly prose, will be expected by those familiar with his writings. They combine to make this one of the best non-specialist books on the subject that the reviewer has seen.

The book is well made and printed, and there are comparatively few printer's errors. One or two minor slips, which might be misleading to the lay reader, should be mentioned so that they may be corrected in later editions. On page 72 the reference to " a smaller number whose mass was only 35 units " should read " a larger number . . . .". On page 148 the statement that the lightest uranium isotope has mass 234 should be qualified by describing it as the lightest naturally occurring uranium isotope, otherwise the later reference to U233 may be confusing. On the same page, a few lines further on, it is said that there are 82 protons in the nucleus of Ba138 ; this should read 82 neutrons. K. E. B. JAY.

Electrons (+ *and* −), *Protons, Photons, Neutrons, Mesotrons and Cosmic Rays*, by Robert A. Millikan. Pp. x+642. (London and Chicago: The University of Chicago Press, 1947. Agent: Cambridge University Press.) Second Edition. 30*s*. net.

The present volume is a revised edition of the 1936 issue, in some respects brought up to date. It is by no means as comprehensive as the title makes one believe since many important aspects connected particularly with neutrons and protons are not even mentioned. The book might be used by a student for preparation of his final examination, but he would certainly fail if it was the only source of information. In spite of this it strikes me as a very valuable contribution, but ought to be called " Professor Millikan's Contributions to Modern Atomic Physics ". With this title it is even more impressive, as it shows how much we owe to Millikan. The historical approach as seen from the Norman Bridge Laboratory of the California Institute of Technology in beautiful Pasadena is very instructive; the arguments which form the basis of the present ideas of atomic physics, too often taken for granted by the modern student of physics, are mostly presented in a dramatic manner. This and the complete absence of mathematics make the book very readable even for non-physicists (the appendix contains some derivations of formulae used in the text; the little space devoted to this only permits to make a result plausible, but certainly gives no hint of theoretical background). It is a little surprising to find that there are still a number of free positive electrons in the nucleus (appendix F, p. 620).

Picking out for special consideration chapter XVI on the release and utilization of nuclear energy, one can expect some strong comments from Professor Millikan. Section 1 on the packing fraction is devoted to the origin of solar energy production and of that of the cosmic rays, section 2 to radioactivity and fission. The real fun begins with section 3 on the control of the atomic bomb, a section which includes very pertinent considerations on the use of atomic power. Millikan does not believe that atomic energy can ever become a major source of energy because the processing cost of nuclear fuel is very high, whilst coal is ready for use. Moreover, he makes the point that to use " atomic packing fraction energy " is very objectionable as it involves using up rare and valuable elements such as U and Th. He says : " In any case, intelligence demands the conserving, rather than burning up, of a very rare and precious resource ". As far as the atomic bomb is concerned, Millikan says : " The great service to mankind of the advent of the atomic bomb has been to make as clear as crystal, to all classes and conditions of men the world over, the necessity to put a stop to international war ".

Chapter XX contains, in some 60 pages entitled the Atom-Annihilation Hypothesis as to the Origin of the Cosmic Rays, much of Millikan's cosmic ray experiments and speculations in a very persuasive form.

It would be easy to throw stones and even heavier objects at the author when one goes into a discussion of historical justice. I am not qualified for this, and at any rate would consider it very unfruitful to enter into the jungle of controversy. E. BRETSCHER.

Cosmic Rays and Nuclear Physics, by L. JÁNOSSY. Pp. xiv+186. (The Pilot Press, 1948.) 9*s*. 6*d*.

The " Frontiers of Science " Series, published by the Pilot Press, in which Professor Jánossy's book appears, aims " to set a new standard in popular science . . . . intended for the growing non-specialist public which wants to keep abreast of recent scientific developments . . . . ". To do justice to *Cosmic Rays and Nuclear Physics* this body of readers must be identified from the internal evidence of the book, and the term " non-specialist" must be accepted in a rather rigorous sense. This book may be recommended strongly to all who have recently taken an honours degree in physics; it will be of the greatest value to students now reading courses which include nuclear physics (for as the title aptly suggests, the cosmic-ray physics of today promises to be the father of the nuclear physics of tomorrow), and, indeed, would be essential to all research workers in the cosmic-ray field were it not for the recent publication of Professor Jánossy's more exhaustive work. Those with less specialist equipment are likely to find difficult reading, involving a complexity of theme and demanding a mature if elementary mathematical sense. Their task is made the more difficult by a number of weaknesses in the diagrams and in the setting of the mathematical matter which, while they do not seriously trouble the reader who is familiar with the subject, are likely to be a serious hindrance to others.

The treatment is, on the whole, conservative and conveys admirably the form of argument involved and some of the pitfalls encountered in the cosmic-ray physics of the last decade. The chapters on the Analysis of Cosmic Ray Particles, on the Theory of the Meson and its Instability, and on the Geomagnetic Effects will be particularly stimulating to the serious reader, and all cover excellently matter which has not been well described at this level. The treatment of the Soft Component is less successful—a more physical and less mathematical approach would probably have been more satisfactory—while the chapter on Penetrating Showers is detailed to an extent which the topical interest of these events in cosmic-ray laboratories at the moment cannot completely justify. The treatment throughout, however, has the authority which can only come from close contact with the crucial problems of the subject as they have arisen, and the balance of the discussions of experimental work is very sound.

The illustrations are an unsatisfactory feature of the book. Too many line diagrams are insufficiently described, while a large proportion of the half-tone illustrations are too small to be of value to anybody not familiar with the originals. In particular, the line diagram figure 41 has been allowed to appear over a most unfortunate title. The classical apparatus of C. T. R. Wilson, the functional excellence of which has always been apparent to later cloud-chamber workers, was not remotely like this illustration, which should be replaced in any later edition.

J. G. WILSON.

Obituary Notices

ALEXANDER MÜLLER

DR. ALEXANDER MÜLLER, who died on 2 July 1947 at the age of 58, was well known for his work on x rays. He began his studies in his home town of Zurich, where he specialized in mathematics and physics and was awarded a doctorate for a thesis on "Messungen der thermischen Ausdehnung von kristallisierten Quartz und von Gold zwischen 18° und 540°". Later he worked with Debye in Germany and in 1919 came to Cambridge to work under Lord Rutherford.

By this time his main interest was in x rays. He had already made one of the earliest determinations of the value of Planck's constant, using an x-ray method, and had become interested in x-ray spectroscopy. At Cambridge he continued with his work on x-ray spectroscopy and published a paper on the L spectrum of mercury for which work he designed and made a special x-ray tube with a liquid mercury anticathode, and a photographic spectrometer which, while capable of high accuracy, was a model of compactness. Later he was consulted by Professor G. I. Taylor, who was interested in the slip phenomena of single metal crystals, and he developed a method for the study of such crystals by x-ray diffraction; the results obtained in this way formed the basis of the Bakerian Lecture by Taylor and Elam in 1923. In the course of this work he himself made many sodium and lithium single crystals and experimented on their deformation.

His colleagues at the Cavendish Laboratory were quick to realize that in Müller they had in their midst an experimentalist of outstanding ability and one who combined a sound theoretical knowledge with an acute critical faculty. Although his work was rather apart from the main radioactive researches of the laboratory he was continually being consulted by other workers on their own particular problems.

In 1922 he left Cambridge to join Sir William Bragg's research staff at University College, London, where Bragg was collecting around him a research group whose main aim was the application of x-ray methods in the study of crystal structures. About this time Bragg had just broken new ground by his pioneer work on the structure of naphthalene and anthracene, the first organic structures to be investigated by x-ray methods. Up till then Bragg and his colleagues had used for the most part the Bragg ionization x-ray spectrometer. Müller brought to the work his extensive knowledge of photographic methods, and it was not long before such methods were being used to supplement the ionization methods which still had many advantages, especially in the study of single crystals. For powders and crystal aggregates the photographic technique was both an easier and a more reliable method of investigation.

In 1923, Bragg was appointed Director of the Royal Institution and took with him from University College a team of workers to provide a nucleus of a Research school in the Davy Faraday Laboratory attached to the Royal Institution. Müller was one of this team and it was destined that the rest of his life's work should be carried out there.

The writer can well remember his first visit with Sir William Bragg to the Laboratory. It was very nearly deserted; in most rooms there was a collection of discarded and broken apparatus, accumulations of stones, presumably the object of study of some bygone mineralogist, and similar débris. There was no equipment of the type required and the workshop facilities were of the poorest. This was soon changed, and in the fitting out of the Laboratories and workshop Müller played an important part. He threw himself with his infectious enthusiasm and zeal into the task, and before long a collection of workmanlike laboratories was available and a new workshop was turning out first-class spectrometers and other equipment. Even in those early days Müller arranged for a vacuum service to be laid on to the various laboratories.

There was none too much money available for research in those days and at any rate there was little of the specialized x-ray equipment required that could be bought in the open market. Müller's contribution in overcoming these difficulties was a notable one. He made his own high tension coils, interruptors, high vacuum pumps, and X-ray tubes, and taught others to be equally self-reliant, although few of his colleagues ever got quite as high efficiency from their apparatus as did Müller. Still, with Müller at hand to help, and he was always ready and willing to do so, they produced results.

His best known researches at the Davy Faraday Laboratory were connected with the structure of long-chain organic compounds. Just as Bragg had opened up a new field in his study of the aromatic hydrocarbons, so Müller, at first in collaboration with Shearer and Saville and later on his own, broke new ground by his studies of the aliphatic compounds. His work on the normal fatty acids and hydrocarbons (published in the *Journal of the Chemical Society*, 1923–25) established for the first time the dimensions and details of structure of such molecules, and this work was elaborated later when Müller succeeded in growing single crystals of some of these compounds, which enabled details of structure to be determined which could not be elucidated by studies of the highly oriented powders used in the earlier investigations. His later researches on these materials, including work on the Van der Waals forces, lattice energy, dielectric polarization and compressibility, were published in a series of papers presented to the Royal Society between 1928 and 1941. At the same time he was taking an active part in directing and supervising the researches of the large group of research workers in the Davy Faraday Laboratory of which in 1928 he had been made Assistant Director.

Another important contribution made by Müller during these years was the development of high power X-ray tubes. It was becoming increasingly important to be able to obtain X-ray diffraction photographs of substances in short times, especially for the study of thermal changes, elastic properties and the like. Müller designed and had made the first really efficient high power X-ray tube. This could deal with a continuous input of 5 kw. and was of the continuously evacuated high vacuum type with a water-cooled rotating anticathode. There were many difficulties in the construction of such a tube, mainly concerned with the maintenance of a high vacuum and the removal of the heat generated by the discharge, but these Müller with his customary ingenuity overcame. Tubes of this type were in continuous operation at the Davy Faraday Laboratory for many years, being used, for example, in connection with researches on the thermal vibration of atoms in crystals and on elastic properties of hydrocarbons. Later, in conjunction with Clay, he embarked on a much more ambitious project, the construction of a 50 kw. tube. This was an engineering proposition and involved not only the design and construction of the tube with its huge rotating anticathode some two feet in diameter, but also a three-phase generator using continuously evacuated rectifying valves. This task was successfully completed and an account of the outfit was published in the *Journal of the Institution of Electrical Engineers* in 1939.

During more recent years Müller's health left much to be desired, but in spite of this he carried on cheerfully with his work and was involved in various researches connected with problems arising out of the war.

Müller will be remembered with affection by a large number of workers who passed through the Davy Faraday Laboratory for the kindly way in which he assisted in their researches and for the valuable advice always so willingly and efficiently given and so often enlivened by a touch of humour. Not only was he a very sound physicist and a more than adequate mathematician, but he had wide scientific interests. It is probable indeed that had he been given the option of choosing his career again he might well have decided to study the more physical aspects of biological science. He had many other gifts. He was a first class linguist; English, French, German or Italian he spoke with nearly equal fluency. He was interested in art and music, could perform adequately on many instruments and was a good singer with an amazingly wide repertoire of songs of many countries. Many who made the trip across Canada with the British Association in 1927 will remember the impromptu concerts on the train and Müller's performances, instrumental and vocal. He was a keen mountaineer and spent most of his holidays in his native mountains, although he had also in his heart a warm spot for the hills of Skye.

He was elected a Fellow of the Physical Society in 1925.

G. SHEARER.

SIR CLIFFORD COPLAND PATERSON, O.B.E., D.Sc., F.R.S.

THE death of Clifford Copland Paterson at the age of 68 removes a personality from our midst who has had a notable influence on the application of the developments of physical science to technological purposes.

After being at Mill Hill School in the time of the Headmastership of Sir John McClure, he was at the Finsbury Technical College and at Faraday House when it was under the guidance of Alexander Russell, of whom he always spoke with admiration and affection. He worked for some time with George Wailes & Co., General Engineers, and he supplemented his training at Faraday House in the works of the Maschinen Fabrik Oerlikon, near Zürich, which gave him a valuable experience of the practical side of electrical engineering. The leading Swiss engineering firms were among the most capable in Europe, no doubt due, in part, to the training available at the Technische Hochschule at Zürich, which young British engineers used to attend a generation previously, as there was no institution of similar standing in this country. During this period he worked for the firm in erecting the electric equipment in the tunnel of the Jungfrau Railway, an experience which gave him opportunities for walking and climbing in Switzerland ; this meant to him, as to many others, a renewal of health of body and spirit which few experiences can equal. This varied training in general engineering served him in good stead in his later life in giving him an intuitive basis for the guidance of himself and others as to how the practical requirements of a problem might best be tackled.

Soon after the foundation of the National Physical Laboratory he was chosen by Dr. R. T. Glazebrook, the Director, to undertake the technical side of electrical work. The more fundamental researches, including the determination of the values of resistances and standard cells in terms of the "absolute" C.G.S. units, and of inductance and capacitance, were in the hands of F. E. Smith and A. Campbell. The most important part of Paterson's early work was the development of equipment for determining the accuracy of commercial types of apparatus submitted to the Laboratory for verification, including voltmeters, wattmeters and ammeters for alternating current. The present writer, after engineering experience in Manchester and Rugby, joined the Laboratory Staff soon after Paterson's appointment, and the design and installation of what has proved to be the most accurate and flexible system in any national laboratory for the measurement of alternating current quantities was one of the results of several years of close collaboration. The essential instruments, a voltmeter and a wattmeter, both of the electrostatic type, were developed so as to provide an accuracy of a few parts in 10,000. In those early days there was no public electric supply available at the Laboratory, and one remembers having to test three-phase instruments when the only power supply available was an overloaded gas engine.

Another branch of the electrotechnical work was the development of accurate measurements of candle power of light sources and the design and installation of standards of adequate luminous stability. The "Vernon Harcourt" Pentane Lamp was one of the recognized standard lamps in use, especially in the gas industry, and as it had also a nominal value of about 10 candle power, which was a more convenient size than working standards of about 1 candle power, it was adopted as a reference standard. As it was a flame lamp, the illumination from it was dependent on atmospheric conditions, particularly barometric pressure and humidity, and the influence of these had to be accurately determined. Moreover, it was found that lamps varied appreciably in the amount of light which they emitted, partly due to small variations in methods of construction. This had been only partly recognized previously, as the recently introduced contrast photometers were capable of detecting smaller differences between light sources than earlier types of instrument. In those days the general source of domestic illumination was the carbon-filament lamp, requiring about 4 watts per candle, compared with modern lamps of 1 watt and less. B. P. Dudding, J. W. T. Walsh and W. A. Barnett were closely associated with Paterson in this work, the development of which has continued to the present day, with the requirements imposed by the evolution of more efficient and "whiter" sources of light, involving the comparison of light of different colours.

In later years Paterson and the Staff of the Laboratories of the General Electric Co., gave most valuable and willing assistance to the Staff of the N.P.L., providing lamps specially designed as photometric standards, the construction of which required the technique and manufacturing methods only available in a lamp factory.

This led to the accurate intercomparison of the relative size of the unit of light in use in other countries—America, France and Germany—by circulating lamps among their national laboratories. The next step was to adopt as the recognized unit of candle power such a value that a lamp measured in one country would be assigned the same value when tested in another ; this was largely brought about by the efforts of the International Illumination Commission, of which Paterson was one of the most influential members.

This was followed by the decision to develop an International Standard which should not be dependent upon electric lamps which might change slowly or be destroyed ; and during the inter-war years it was left to the national laboratories of England and America to collaborate on a more fundamental unit dependent upon the brightness under " black-body " conditions of a surface at the melting point of platinum. The necessity for high accuracy made this a difficult piece of work, and success in this country was largely due to H. Buckley.

In all this work, some of it of a semi-political character, involving in some countries the supersession of long established standards, Paterson's support was very helpful. The international recognition of this work was delayed for several years by the war, and it was only on 1st January 1948 that the " New Unit " came into official use at the same time as the " absolute " C.G.S. units of electrical quantities which superseded the previous empirical " International " Units, forty years after the change was urged by this country and America at the International Congress of 1908, but which did not receive unanimous support.

The work of Paterson and others at the National Physical Laboratory established the science and art of photometry on a sound basis in this country. By discussing the various problems involved with instrument makers and lamp manufacturers difficulties were gradually overcome.

During the war, 1914–1918, the results of development work in other countries, particularly Germany, were no longer available. It was a period of rapid change in the material used in lamps, carbon giving way to tantalum and tantalum to tungsten.

It was obvious that more research and development work was required in this country ; and Paterson's advice as to how this should be initiated was sought by the Directors of the General Electric Co. As a result of the discussions of the organization which he suggested, he was invited to undertake the establishment of a Research Organization which he eventually accepted, resigning from the N.P.L. at the end of 1918 and undertaking his new duties on 1st January 1919.

National work of some importance which arose shortly before Paterson retired from the N.P.L. to join the General Electric Company was in connection with a committee appointed by the Board of Trade to advise on Navigation Lights of Merchant Ships at sea. International general rules for navigation lights had been in operation for some time, but no detailed technical specification of performance of the various types of lamps had been produced. The principal requirements were to determine the effective candle power required to enable an oil lamp to be seen at a distance of two miles in the case of the coloured starboard and port lights, and at five miles in the case of the mast-head lamps. A lens system had to be designed to enable this to be attained, providing adequate vertical divergence of the beam to enable a ship's light to be seen when heeled over several degrees, a factor of great importance in the case of sailing ships. There were also the colour intensities of the red and green lights to be specified. The writer was a member of the committee and carried out with Messrs. Walsh and Buckley most of the investigatory work involved. Paterson became Chairman of the Committee in its later stages and showed in these comparatively early days that, by his clearness of exposition and friendly appreciation of the views of others, sometimes without much foundation of a scientific character, he could persuade them without difficulty to agree with his proposals when he considered them important.

He was Joint Secretary with D. R. Wilson (later Sir Duncan Wilson) of the Home Office Committee on the Lighting of Factories and Workshops which made its Report in 1915 ; and he was later Chairman of the Committee of the Building Research Board on the Lighting of Buildings.

An influential side of Paterson's activities was his association with the professional and technical organizations, British and foreign. He and the writer carried out work in the early years of the N.P.L. on behalf of the Engineering Standards Committee which later developed into the British Standards Institution. One of his early tasks in this direction was the investigatory work on the performance of commercial electric lamps, the results of which formed the foundation of the original specification of performance of (carbon) filament lamps. Since those days, up to the present year, we have been continuously associated with the work of the Electrical Industry Committee of the Institution and of its various Sub-Committees. Paterson was Chairman of this Committee for some years until he was elected Chairman of the Engineering Divisional Council and later Chairman of the General Council of the Institution. His election to such important positions was an indication of the appreciation of his friendly advice and of his understanding of different points of view as well as of his knowledge of engineering. His services in maintaining the quality of the work of the B.S.I. have been put on public record by the Director, Mr. Percy Good.

An indication of his activities and of the esteem in which he was held is shown by the fact that he had been President of the Institute of Physics, the Illuminating Engineering Society, the Junior Institution of Engineers, the Institution of Engineers-in-Charge and the Electrical Research Association. He was elected President of the Institution of Electrical Engineers for the year 1930–31, during which the Centenary of Faraday's discovery of electromagnetic induction, the foundation on which all the technical applications of electricity have been developed, was celebrated on an international scale. No happier choice could have been made.

On this, and many other occasions, he showed that he was an unusually effective lecturer, particularly to audiences of a popular character. He would describe physical phenomena in simple language, illustrate them by analogies which could be easily understood and proceed to the experiment which illustrated the phenomenon under discussion. He might refer to the transmission of speech by using radio waves or waves of light as the carrying agent, or the history of various forms of sources of light, or one of many other applications. The audience was sure to have both a widening of their knowledge and an intellectual treat of absorbing interest.

Reading through the scripts which he used at his most recent lectures one is struck by the marks in nearly every line indicating the varying degree of emphasis to be used in order to render it as intelligible as possible to the audience.

Among the many papers and addresses which he gave, those contributed, some of them jointly, to the Physical Society, were: 1909, International Standard of Light; 1912, Point Sources of Light; 1915, Temperature and Colour Identity; 1915, Unit of Candle Power in White Light; 1932, Addresses on the use of Photo-cells; 1937, Guthrie Lecture, Appraisement of Lighting. He was a member of the Council of the Society from 1914–1919 and Treasurer from 1938–1946.

Instances of his more recent contributions of an influential character are an Address to the Parliamentary and Scientific Committee, a paper contributed to the Stockholm Congress on Management, the lecture at the Central Hall on "The Electron Liberated", on the occasion of the fiftieth anniversary of the discovery of the electron by J. J. Thomson, and a lecture to the Royal Society on the "Research Laboratories of the General Electric Co." These were given in 1947, shortly before his departure with Lady Paterson for some months to Australia, where he was assured a cordial welcome as the founder of a Research Laboratory of world-wide reputation, as an authoritative exponent of British schemes of standardization which have extended their scope far beyond what was dealt with even a dozen years ago, and also as a Director of an English engineering firm, employing some 50,000 people, which has close contacts with all parts of the Commonwealth and with many other countries. Not least was it an opportunity for many to meet one whose staff had been largely instrumental in all the developments, production and utilization of equipment required for various forms of radiolocation.

The Twenty-First Anniversary of the founding of the Research Laboratories was celebrated in February 1944 with such festivities as war permitted. Among the events was a lecture by Paterson on the "Confidential History of the Research Laboratories" in which he described their foundation, growth and activities. The lecture provided a fascinating account of organization and design and of what he considered were the proper relations which should be established between the Laboratory and the various departments of the Company which it was to serve.

The present writer, as one of his oldest friends, had the pleasant task of proposing the thanks of the meeting and in doing so informed the audience that an abiding memory of Paterson of thirty years ago was that he always carried a centre-punch in his pocket. To everyone's surprise Paterson stood up and produced a centre-punch, showing that though, as he sometimes lamented, he had no time to do practical work himself, yet an elementary engineering tool did at least provide a tangible memento of his earlier joys in handicraft.

Two events occurred during recent years which enabled the Staff of the Laboratories to show their friendly affection for him and their appreciation of his services. One was his election as a Fellow of the Royal Society. In an address to the Staff on that occasion he said how glad he was that this had happened, as he regarded it primarily as a recognition of the value of their work over many years in widening the knowledge of science and developing its applications. The other was the knighthood which had been bestowed on him. He was in doubt as to whether he should accept the honour and, when his intimate friends persuaded him how disappointed the Staff would be if they learnt that he had declined, he decided to accept. The reception of the news at Wembley can have left him in no doubt as to the rightness of his decision.

In his relations with his Staff he regarded himself as a fellow traveller in life's journey, always giving service as he expected it of them. He " clocked in " at the beginning of the day's work, as was the duty of all the rest of the staff, and in his daily contacts with all and sundry he showed that he regarded them as colleagues in the important labours on which they were engaged.

Paterson was ready to take bold decisions in the national interest. At one stage in the war he and his Senior Staff realized that large numbers of novel types of valves would very likely be suddenly required. It was realized that the usual routine procedure for obtaining orders, particularly of a speculative character, would cause too serious delays ; and production groups were set up in the Laboratories, employing eventually some 700 women, who were taught the various technical operations of glass blowing, making glass-to-metal seals, cathode formation and many other arts, including the production of a very high vacuum and sealing-off.

Eventually the day came when operational and supply officers did require these valves very, very urgently, and did not know how to get them. Paterson disclosed his private store of trained skill and valves, and the relief of the Services can be imagined. He had saved six to eight months of training and development which would otherwise have been required for the provision of valves which were essential for the national safety at a most critical period of the war, more essential indeed than anyone originally imagined would be the case.

In the course of his address to the Staff in February 1944, already mentioned, Paterson gave a short outline of the development during the war, and in conclusion he said :

> " Wherever one opens the book of our war activities there one finds a fascinating story of apparently impossible things accomplished and difficulties surmounted by the patience and determination of individuals drawn from all parts of the Laboratories and Shops, backed by girls and women in the Offices and Laboratories and working in Groups unselfishly for the one object. It is true that this has always been the tradition of the place from its commencement ; but the 1000 newcomers to the Laboratories since 1939 had not had the years of apprenticeship of the older members nor, in pre-war times, have people had the appalling irritations and constant discouragements arising from Government controls, blitzes and other man-made hindrances to rational existence. Therefore I return to where I started in this talk, and thank Providence which has permitted me to share the comradeship of such a staff through the days of peace and still more wonderful days of war. I trust it is not too much to hope that I may still tread with you the adventurous paths of reconstruction, whenever our war task shall draw to a close."

It was hoped by his friends that the new experience of his journey to Australia, which commenced in the early days of 1948, would provide him with a well-earned holiday, in spite of a full programme which had been prepared for him, and that he would return with renewed zest to " tread with them the adventurous paths of reconstruction ". However, it was not to be. He became seriously ill in Australia, surviving the return journey only a few days, and died on 26th July.

He was awarded the O.B.E. for his work with Walsh in developing an electrically indicating aircraft height finder in 1916–1917, and was made an Honorary Doctor of Science of Birmingham University in 1937. In 1946 he was given the James Alfred Ewing Medal, which is awarded jointly by the Royal Society and the Institution of Civil Engineers, and in 1948 he was the first person who was not an American to be awarded the Gold Medal of the Illuminating Engineering Society of America. In the summer of 1948 he was elected as Honorary President of the International Illumination Commission, which has distinguished no other person in this manner.

It is hardly possible, in giving an account of Paterson's life work, not to include Lady Paterson in one's thoughts and esteem. It was characteristic of her to insist, with her daughter, in joining the Laboratory Staff as a volunteer and beginner in valve construction and assembly when urgency for output was becoming very serious. She carried on this work for many months.

Her presence and kindly interest in the members of the staff was a continual source of encouragement and sympathy, and must have been a reassuring influence when many were engaged on delicate manipulative work, often interrupted by air raid warnings. At one period these warnings were to be expected daily, and the A.R.P. record shows that on one day ten were received and circulated to the staff.

Lady Paterson took charge of the War Savings Organization of the Research Laboratories, the funds of which have amounted to £110,000.

The mutual affection between Sir Clifford and Lady Paterson is hardly a subject which one can discuss, except to say that it was a living witness of two characters which can be an inspiration to all.

E. H. R.

DR. ALLAN FERGUSON writes: May I be permitted to add an account of two incidents which seem to me very characteristic of our old friend?

The first deals with an evening address which Paterson gave at a York meeting of the British Association in 1932. The address had drawn to its close and a spontaneous burst of applause was dying away when the enthusiastic voice of the President of the Association, Sir Alfred Ewing, was heard to ring out: " Oh, you incomparable showman." And without the slightest *arrière pensée* the President's almost instinctive shout of delight was a testimonial to Paterson's supreme craftmanship. Yes, indeed, he was a great artist, with the art that conceals art. Every experiment marched to its appointed close with the utmost precision, and Paterson had no need to look at the lantern screen in order to see what was thereon. Paterson's task in any address which he gave was to make sure that the facts and principles should be brought out in the most telling fashion, and he never failed in this task.

The second incident, which illustrated another side of Paterson's character, occurred only a few months ago. We both arrived early for a Committee meeting and were talking of the world situation when I remarked that I could see no light anywhere. " Well ", said Paterson, " there is at least a possibility of light ". " In what ? " " In a return to Christian ethics and Christian principles."

EMERITUS PROFESSOR S. W. J. SMITH, M.A., D.SC., F.INST.P., F.R.S.

SAMUEL WALTER JOHNSON SMITH was born at Ferry-Port on Craig, Fife, in 1871, and was the second son of Walter Mackerzie Smith and of Margaret Black. His father was a pioneer in railway development in Japan and was responsible for many improvements in locomotive design, notably in connection with the compound steam engine. He was educated at Tayport Public School, West End Academy, Dundee, and Rutherford College, Newcastle-on-Tyne. From Rutherford College he won a National Scholarship to the Royal College of Science in 1887 which he held until he went up to Trinity College, Cambridge, with a Major Scholarship, in 1891. He obtained a first class in Part I of the natural science Tripos in 1892 and a first class in Part II in 1894, in both Physics and Chemistry. A first class in the two subjects is an unusual achievement and this equipped him for the two distinct branches of research which he later developed.

He became Coutts Trotter Student and began work in the Cavendish Laboratory under J. J. Thomson, choosing as his subject electrocapillary phenomena and the capillary electrometer. Smith, who was discriminating in his praise, always spoke with the greatest respect and admiration of Thomson and his work. This early interest in electrolytic effects led to one of his major fields of investigation. He published a paper on "The nature of electrocapillary phenomena" in the *Philosophical Transactions of the Royal Society* in 1899; and a paper on "A portable capillary electrometer" in 1903, the first of a long series in the *Proceedings of the Physical Society*.

Smith had returned to the Royal College of Science as Senior Demonstrator in Physics in 1897. Sir Arthur Rücker, who was then the Professor of Physics there, suggested to him that an investigation of magnetism in meteoric iron might shed light on the nature of the earth's magnetism. This suggestion led to Smith's first work on magnetism which developed into his deep interest in the magnetic properties of materials, an interest which he retained throughout his life. An account of his early experiments is given in the *Philosophical Transactions of the Royal Society* of 1908 in a paper entitled "The thermomagnetic analyses of meteoric and artificial nickel iron alloys". He at the same time continued his work on electrolytic phenomena which was published in a number of papers in the *Proceedings of the Physical Society* of about that period. H. Moss and W. F. Higgins collaborated with him in parts of this work.

In 1912 Smith was made an Assistant Professor in what had now become the Imperial College of Science and Technology. This was a period of rapid and most fruitful development of his two main lines of investigation. His interest in magnetism and its dependence on temperature had now extended to the carbon steels and their constituents. Papers in the *Proceedings of the Physical Society* followed each other in rapid succession; "The magnetic transition temperature of cementite" (with W. White and S. G. Barker) 1911; "The self-demagnetization of annealed steel rods" (with J. Guild) 1912; "The thermomagnetic study of the eutectoid transition point of carbon steels" (with J. Guild) in the *Philosophical Transactions of the Royal Society* of 1914. In that year, 1914, he was made a Fellow of the Royal Society. His interest in ionic problems continued and in 1916 he published three papers dealing with this subject in the *Proceedings of the Physical Society*.

The conditions under which Smith was working at the Imperial College were particularly congenial to him; he was in charge of the advanced teaching laboratory and enjoyed talking to the relatively small and keen class of about twenty students. He was free from departmental administrative duties, and had ample time for his research work, with facilities and space to carry it out. His wholehearted devotion to his subject could be indulged to the full. He was on the most friendly terms with H. L. Callendar, who was now professor, and his family. This happy atmosphere made a lasting impression on him. In Smith's own words, "Callendar's friendliness and entire freedom from affectation of any kind, with no hint of superiority in anything he said or did, made a deep impression on everyone who knew him".

In 1919 Smith was appointed to the Chair of Physics at Birmingham. One of his first duties was to organize the teaching of a full honours school in the subject. This meant the introduction of new advanced courses in which he took a very large share. In his first few years at Birmingham he also undertook a good deal of the more elementary lecturing, particularly in electricity and magnetism, including the large first-year class. These first few years coincided with the return of large numbers of ex-service men and women from the first world war. This necessitated an extension to his department by the addition of laboratories housed in wooden huts. In all these developments he took a leading and active part.

As a teacher he was careful and exact. He enjoyed the development of a physical argument and to do this he always prepared detailed lecture notes which he wrote and re-wrote until the exact wording was to his liking. This meant that to him lecturing was no easy matter, and his method was probably most appreciated by the more advanced students to whom he lectured on thermodynamics and crystal optics, with also a complete course in the more recent advances in physics. His attitude to the announcement of new discoveries was cautious; he did not wish to appear fashionable or at the head of the field in such matters. He was, however, a great deal more interested and knowledgeable in the progress of new ideas than he was generally prepared to admit, and anyone would have been foolish to assume his ignorance of such things. When new ideas had been satisfactorily established they quietly appeared without delay in his own teaching.

Although as head of the large physics department at Birmingham he had a good deal of teaching and administrative work to do, his main interest still lay in his research. He now concentrated on magnetism and on the other properties of metals with which magnetism is associated. In this connection he recognized the power of the new techniques provided by micro-photography and by x-ray crystal analysis and work was begun on both these aspects of his problem. In this he had the devoted assistance of two of his colleagues, A. A. Dee and the late James Young, who threw themselves energetically into the work. Dee concentrated on the thermo-magnetic analysis and the micro-photography, while Young developed the work with x rays. This extension of the research to the crystallographic side remained one of Smith's main interests, the problems of solid geometry which were involved being a source of delight to him. In 1924 he published in collaboration with A. A. Dee and M. V. Mayneord " The magnetism of annealed carbon steels " in the *Proceedings of the Physical Society*. In 1928 an important paper in three separate parts on " The mode of formation of Neumann bands " dealing with the later developments of the work, was published in collaboration with Dee and Young in the *Proceedings of the Royal Society*.

In addition to his normal duties as a professor, Smith voluntarily undertook while at Birmingham a number of additional responsibilities in connection with his department. These included an active part in government investigations on radio direction finding (under the Radio Research Board of the Department of Scientific and Industrial Research), and on the transmission over long distances of the sound of gun-fire with Dr. Guy Barlow (under the Meteorological Office). At the request of Sir Gilbert Barling, the Pro-Chancellor, on behalf of the National Radium Commission, he established a laboratory to provide radon for medical use throughout the Midlands. In all such matters he took a deep interest. He held the office of Dean of the Faculty of Science from 1929 to 1932 with dignity and distinction. Although he did not claim to be, or wish to be, a man of affairs, he brought to the office a fund of shrewd judgment and wide learning, and an extensive knowledge of University matters.

Smith retired from the Chair of Physics at Birmingham in 1936 and was made an Emeritus Professor of the University. He continued his interest in the magnetic and crystal problems, his last published work being a note, with Young, to *Nature* in 1929. " The Widmanstätten structure of octahedral meteoric iron ". The subject on which Smith was working had become a highly complex one and it was no easy matter to correlate and explain the large mass of experimental data which he and his collaborators had collected. In his later years failing health prevented his completing this work and it was a matter of regret to him that the results of the work of a number of those who had helped him should not have been published.

Smith always took a special interest in the work of the Physical Society and held the office of Secretary from 1908 to 1916, and that of Vice-President from 1916 to 1919 when he left London for Birmingham. In later years his affection for the Society continued. One of the first meetings of the Society outside London was held in Birmingham in 1929 when a number of papers were read on a wide variety of subjects dealing with his own work and with other investigations which he had freely encouraged in his department. This was a very happy occasion for him.

Many who knew him will remember him for his friendliness and kindness. He was no respecter of personages but enjoyed talking to simple folk and especially to children. In these talks he would occasionally embarrass his young friends by asking questions on science or arithmetic. Smith liked to tell the following little story against himself. He had been asking a very youthful porter on a suburban station a number of such questions, when the boy neatly and politely extricated himself by saying " I think your train is coming now, sir ".

He died on the 20th August 1948. In 1900 he married Dorothy Muriel Stamp. His widow and his daughter Jean survive him.

T. L. IBBS.

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